1) Let $M$ be an $n$-manifold and $F : M \to M$ a diffeomorphism. The suspension of $F$ is defined by taking $M \times [0,1]$ and identifying every point $(x,0), x \in M$ with $(F(x),1)$, and is denoted by $M_F$.

(a) Show that $M_F$ is an $(n+1)$-manifold

(b) What is $M_{Id}$?

For the next problem we will use complex numbers $\mathbb{C}$. A complex number is of the form $z = x + iy$, with $x, y \in \mathbb{R}$ and $i$ the imaginary unit. Multiplication of complex numbers is defined in the usual way together with the rule that $i^2 = -1$. Mapping $z$ to $(x, y) \in \mathbb{R}^2$ we obtain thus an identification of $\mathbb{C}$ and $\mathbb{R}^2$ as real vector spaces. The absolute value on $\mathbb{C}$ is defined by $|z| = \sqrt{x^2 + y^2}$, which is the same as the usual norm in $\mathbb{R}^2$. Similarly, $\mathbb{C}^n$ is the vector space of $n$-tuples of complex numbers $(z_1, \ldots, z_n), z_j \in \mathbb{C}$, which is identified with $\mathbb{R}^{2n}$ as real vector spaces.

2) Given a vector $a = (a_1, \ldots, a_n) \in \mathbb{N}^n_{>0}$, let

$$V(a) = \{(z_1, \ldots, z_n) \in \mathbb{C}^n \mid |z_1|^2 + \cdots + |z_n|^2 = 1, \ z_1^{a_1} + \cdots + z_n^{a_n} = 0\}.$$

Prove that $V(a)$ is a smooth manifold of (real) dimension $2n - 3$. 
