Throughout this problem set, $V$ will denote a finite-dimensional vector space over $\mathbb{R}$.

1) Consider an element $t \in V \otimes V$, which is identified with bilinear map $\ell_t : V^* \times V^* \to \mathbb{R}$.

(a) Assume that $\dim V \geq 2$, and show that if $t$ is decomposable (i.e. $t = v_1 \otimes v_2$ for some $v_1, v_2 \in V$) then $\ell_t$ is degenerate, in the usual sense that there is $0 \neq \varphi \in V^*$ such that $\ell_t(\varphi, \psi) = 0$ for all $\psi \in V^*$.

(b) Use part (a) to find a nondecomposable $t \in V \otimes V$ for $V = \mathbb{R}^2$.

2) Show that every element in $\Lambda^2 \mathbb{R}^3$ is of the form $v_1 \wedge v_2$ for some vectors $v_j \in \mathbb{R}^3$.

3) Show that

$$T^2(V) = \text{Sym}^2 V \oplus \Lambda^2 V.$$

4) Let $(e_1, e_2, e_3)$ be the standard basis of $\mathbb{R}^3$. Show that the contravariant 3-tensor $e_1 \otimes e_2 \otimes e_3 \in T^3(\mathbb{R}^3)$ does not lie in $\text{Sym}^3 \mathbb{R}^3 \oplus \Lambda^3 \mathbb{R}^3$. 