1) Define a relation $\sim$ on $\mathbb{Z} \times \mathbb{N}_{>0}$ by declaring

$$(x, y) \sim (x', y') \iff xy' = x'y.$$ 

Verify that $\sim$ is an equivalence relation.

2) [Exercise 0.3.10] Let $f : A \to B$ and $g : B \to C$ be functions.

(a) Prove that if $g \circ f$ is injective, then $f$ is injective.

(b) Prove that if $g \circ f$ is surjective, then $g$ is surjective.

(c) Find an explicit example where $g \circ f$ is bijective, but neither $f$ nor $g$ is bijective.

3) Let $A, B$ be finite sets, and suppose that there exist injective maps $f : A \to B$ and $g : B \to A$. Prove that $f$ and $g$ must be bijective.

4) [Exercise 0.3.21] Suppose $A \subset B$ and $B$ is a finite set. Prove that $A$ is finite, i.e. either $A = \emptyset$ or else there is a bijection $f : A \to \{1, \ldots, n\}$ for some $n \in \mathbb{N}_{>0}$. 