

### Homework 1

1. For each  $n \in \mathbb{N}$  find an ideal  $I_n \subset \mathbb{C}[x, y]$  which needs  $n$  generators.
2. Find a finite set of generators of  $I := \langle x^{2n} - y^{3n} : n \in \mathbb{N} \rangle \subset \mathbb{C}[x, y]$ .
3. Consider the lines  $\mathfrak{l}_1 : x_2 = x_3 = 0$ ,  $\mathfrak{l}_2 : x_1 = x_3 = 0$ ,  $\mathfrak{l}_3 : x_1 = x_2 = 0$  in  $\mathbb{A}^2$ . Find generators of the ideal

$$I := I(\mathfrak{l}_1 \cup \mathfrak{l}_2 \cup \mathfrak{l}_3) \subset k[x_1, x_2, x_3].$$

4. Let  $I \subset \mathbb{C}[x_1, \dots, x_n]$  be an ideal. Show that the ring  $\mathbb{C}[x_1, \dots, x_n]/I$  is noetherian.
5. Show that the power series ring  $\mathbb{C}[[x_1, \dots, x_n]]$  is noetherian.