

Homework 8

1. Find all integer solutions of $y^2 = x^3 + 1$.

2. Show that

$$\log(\zeta(s)) - \sum_p \frac{1}{p^s}$$

remains bounded for $s \rightarrow 1 + 0$.

3. Let χ be a nontrivial character of $(\mathbb{Z}/p\mathbb{Z})^\times$. Let $0 < a < b < p$. Show that

$$\left| \sum_{a < n \leq b} \chi(n) \right| < \sqrt{p} \log(p)$$

4. Compute the ideal class group of $\mathbb{Q}(\sqrt{-53})$.

5. Compute the ideal class group of $\mathbb{Q}(\sqrt{11})$.