

## Homework 8

1. Find all integer solutions of  $y^2 = x^3 + 1$ .
2. Show that
$$\log(\zeta(s)) - \sum_p \frac{1}{p^s}$$
remains bounded for  $s \rightarrow 1 + 0$ .
3. Let  $\chi$  be a nontrivial character of  $(\mathbb{Z}/p\mathbb{Z})^\times$ . Let  $0 < a < b < p$ . Show that
$$\left| \sum_{a < n \leq b} \chi(n) \right| < \sqrt{p} \log(p)$$
4. Compute the ideal class group of  $\mathbb{Q}(\sqrt{-53})$ .
5. Compute the ideal class group of  $\mathbb{Q}(\sqrt{11})$ .