

Homework 5

1. Show that $\zeta(0) = -1/2$ and $\zeta'(0) = -\frac{\ln(2\pi)}{2}$.
2. Show that $2B_{2k} = 1 \pmod{4}$, for $k > 1$.
3. Check that $B_{2k} \not\equiv 0 \pmod{17}$ for $k = 1, \dots, 7$, and that there exists a $k \in [1, \dots, 17]$ such that $B_{2k} \equiv 0 \pmod{37}$.
4. Assume that $\sum_{n \geq 1} \frac{a_n}{n^s}$ converges (not necessarily absolutely). Show that $\sum_{n \geq 1} \frac{a_n}{n^{s'}}$ converges absolutely for $\Re(s') > \Re(s) + 1$.
5. Let $\chi(n) = \left(\frac{-12}{n}\right)$. Find all real t such that $L(\chi, it) = 0$.