Homework 4

1. Show that
$$\sum_{n\geq 1} -\frac{\mu(n)}{n}\cdot \log(1-x^n) = x$$
$$\sum_{n\geq 1,\, p\nmid n} -\frac{\mu(n)}{n}\cdot \log(1-x^n) = x + \frac{x^p}{p} + \frac{x^{p^2}}{p^2} + \cdots$$

2. Show that $f(x) = x^p - x - 1 \in \mathbb{Z}_p[x]$ is irreducible.

3. Compute the radius of convergence, in the p-adics, of
$$\sum_{n\geq 1} n! x^n$$

4. Compute the first p coefficients of $E_p(x)$. Show that the convergence radius of the series defining E_p equals 1.

5. Show that for $p \neq q$ the fields $\mathbb{Q}_p$ and $\mathbb{Q}_q$ are not isomorphic.