Homework 3

1. Compute $\sqrt{-1}$ in $\mathbb{Q}_{17}$ to 5 digits.

2. For $c \in \mathbb{N}$ show that the sequence $c_n := c^{p^n}$ converges in $\mathbb{Q}_p$. Let $\gamma = \lim c_n$. Then $\gamma = c \mod p$ and $\gamma^{p-1} = 1$.

3. Prove that every sequence of integers has a subsequence which is Cauchy with respect to $|\cdot|_p$.

4. Let $f(x) := \sum_n a_n x^n \in \mathbb{Q}_p[[x]]$ and let $r = r(f)$ be its convergence radius. Show that $r(f) \leq r(f')$. Give an example where $r(f) < r(f')$.

5. Show that $\mathbb{Q}_p$ admits no automorphisms.