Homework 2

1. Prove that
$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$
is not an integer for any $n > 1$.

2. Prove that
$$x^4 + 4y^4 = z^2$$
has no nontrivial integer solutions.

3. If $a, b \in \mathbb{Z}$ so that neither a, b nor ab is a square then
$$\sqrt{a} + \sqrt{b} + \sqrt{ab} \notin \mathbb{Q}.$$

4. Let $M := \{1, 2, \ldots, 200\}$. Let $N \subset M$ be a subset of cardinality 100 containing at least one number < 16. Then there exist distinct $n, n' \in N$ so that $n \mid n'$.

5. If p is a prime $\neq 3, 7, 23$ then there is a quadratic nonresidue $< \sqrt{p}$.