p-adic numbers / multiplicative functions

- p-adic interpolation: $a^x, \Gamma_p(x)$ (p-adic Gamma function), $E_p(x)$ (Artin-Hasse exponential)
- Dieudonné–Dwork theory: if $f \in 1 + x\mathbb{Q}_p[[x]]$ then

$$f \in \mathbb{Z}_p[[x]] \quad \Leftrightarrow \quad f(x)^p/f(x^p) \in 1 + p\mathbb{Z}_p[[x]].$$

- binomial polynomials: Let $\mathcal{L}$ be the $\mathbb{Z}$-module of all functions $f \in \mathbb{Q}[x]$ such that

$$f : \mathbb{N} \to \mathbb{Z}.$$

  Then $\mathcal{L}$ is free, with basis $\binom{x}{k}$.

- Mahler's theory: Let $f : \mathbb{Z}_p \to \mathbb{Q}_p$ be a continuous function. Put

$$a_n(f) := \sum (-1)^{n-k} \binom{n}{k} f(k).$$

  Then

$$\sum_{k=0}^{\infty} \binom{x}{k} a_k(f) \to f(x),$$

  uniformly.

- Mahler's theory: translation, δ-operators, basis system of polynomials, generalized Mahler series
- Multiplicative functions:
  - divisor function,
  - Moebius function,
  - Euler function,
  - Dedekind ψ-function,
  - von Mangoldt function,
  - Ramanujan τ-function

  and open problems concerning these functions.