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The Life and Work of Gustav Lejeune Dirichlet (1805–1859)

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Dedicated to Jens Mennicke, my friend over many years

Contents

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Introduction

The great advances of mathematics in Germany during the first half of the nineteenth century are to a predominantly large extent associated with the pioneering work of C.F. Gauß (1777–1855), C.G.J. Jacobi (1804–1851), and G. Lejeune Dirichlet (1805–1859). In fact, virtually all leading German mathematicians of the second half of the nineteenth century were their disciples, or disciples of their disciples. This holds true to a special degree for Jacobi and Dirichlet, who most successfully introduced a new level of teaching strongly oriented to their current research whereas Gauß had "a real dislike" of teaching — at least at the poor level which was predominant when Gauß started his career. The leading role of German mathematics in the second half of the nineteenth century and even up to the fateful year 1933 would have been unthinkable without the foundations laid by Gauß, Jacobi, and Dirichlet. But whereas Gauß and Jacobi have been honoured by detailed biographies (e.g. [**Du**], [**Koe**]), a similar account of Dirichlet's life and work is still a desideratum repeatedly deplored. In particular, there exist in English only a few, mostly rather brief, articles on Dirichlet, some of which are unfortunately marred by erroneous statements. The present account is intended as a first attempt to remedy this situation.

1. Family Background and School Education

Johann Peter Gustav Lejeune Dirichlet, to give him his full name, was born in Düren (approximately halfway between Cologne and Aachen (= Aix-la-Chapelle)) on February 13, 1805. He was the seventh[1] and last child of Johann Arnold Lejeune Dirichlet (1762–1837) and his wife Anna Elisabeth, née Lindner (1768–1868(?)). Dirichlet's father was a postmaster, merchant, and city councillor in Düren. The official name of his profession was *commissaire de poste*. After 1807 the entire region of the left bank of the Rhine was under French rule as a result of the wars with revolutionary France and of the Napoleonic Wars. Hence the members of the Dirichlet family were French citizens at the time of Dirichlet's birth. After the final defeat of Napoléon Bonaparte at Waterloo and the ensuing reorganization of Europe at the Congress of Vienna (1814–1815), a large region of the left bank of the Rhine including Bonn, Cologne, Aachen and Düren came under Prussian rule, and the Dirichlet family became Prussian citizens.

Since the name "Lejeune Dirichlet" looks quite unusual for a German family we briefly explain its origin[2]: Dirichlet's grandfather Antoine Lejeune Dirichlet (1711–1784) was born in Verviers (near Liège, Belgium) and settled in Düren, where he got married to a daughter of a Düren family. It was his father who first went under the name "Lejeune Dirichlet" (meaning "the young Dirichlet") in order to differentiate from his father, who had the same first name. The name "Dirichlet" (or "Derichelette") means "from Richelette" after a little town in Belgium. We mention this since it has been purported erroneously that Dirichlet was a descendant of a

[1]Hensel [**H.1**], vol. 1, p. 349 says that Dirichlet's parents had 11 children. Possibly this number includes children which died in infancy.

[2]For many more details on Dirichlet's ancestors see [**BuJZ**].

French Huguenot family. This was not the case as the Dirichlet family was Roman Catholic.

The spelling of the name “Lejeune Dirichlet” is not quite uniform: Dirichlet himself wrote his name “Gustav Lejeune Dirichlet” without a hyphen between the two parts of his proper name. The birth-place of Dirichlet in Düren, Weierstraße 11, is marked with a memorial tablet.

Kummer [**Ku**] and Hensel [**H.1**], vol. 1 inform us that Dirichlet's parents gave their highly gifted son a very careful upbringing. This beyond doubt would not have been an easy matter for them, since they were by no means well off. Dirichlet's school and university education took place during a period of far-reaching reorganization of the Prussian educational system. His school and university education, however, show strong features of the pre-reform era, when formal prescriptions hardly existed. Dirichlet first attended an elementary school, and when this became insufficient, a private school. There he also got instruction in Latin as a preparation for the secondary school (Gymnasium), where the study of the ancient languages constituted an essential part of the training. Dirichlet's inclination for mathematics became apparent very early. He was not yet 12 years of age when he used his pocket money to buy books on mathematics, and when he was told that he could not understand them, he responded, anyhow that he would read them until he understood them.

At first, Dirichlet's parents wanted their son to become a merchant. When he uttered a strong dislike of this plan and said he wanted to study, his parents gave in, and sent him to the Gymnasium in Bonn in 1817. There the 12-year-old boy was entrusted to the care and supervision of Peter Joseph Elvenich (1796–1886), a brilliant student of ancient languages and philosophy, who was acquainted with the Dirichlet family ([**Sc.1**]). Elvenich did not have much to supervise, for Dirichlet was a diligent and good pupil with pleasant manners, who rapidly won the favour of everybody who had something to do with him. For this trait we have lifelong numerous witnesses of renowned contemporaries such as A. von Humboldt (1769–1859), C.F. Gauß, C.G.J. Jacobi, Fanny Hensel née Mendelssohn Bartholdy (1805–1847), Felix Mendelssohn Bartholdy (1809–1847), K.A. Varnhagen von Ense (1785–1858), B. Riemann (1826–1866), R. Dedekind (1831–1916). Without neglecting his other subjects, Dirichlet showed a special interest in mathematics and history, in particular in the then recent history following the French Revolution. It may be assumed that Dirichlet's later free and liberal political views can be traced back to these early studies and to his later stay in the house of General Foy in Paris (see sect. 3).

After two years Dirichlet changed to the Jesuiter-Gymnasium in Cologne. Elvenich became a philologist at the Gymnasium in Koblenz. Later he was promoted to professorships at the Universities of Bonn and Breslau, and informed Dirichlet during his stay in Bonn about the state of affairs with Dirichlet's doctor's diploma. In Cologne, Dirichlet had mathematics lessons with Georg Simon Ohm (1789–1854), well known for his discovery of Ohm's Law (1826); after him the unit of electric resistance got its name. In 1843 Ohm discovered that pure tones are described by purely sinusoidal oscillations. This finding opened the way for the application of Fourier analysis to acoustics. Dirichlet made rapid progress in mathematics under Ohm's guidance and by his diligent private study of mathematical treatises, such that he acquired an unusually broad knowledge even at this early age. He attended the Gymnasium in Cologne for only one year, starting in winter 1820, and then left with a school-leaving certificate. It has been asserted that Dirichlet passed the Abitur examination, but a check of the documents revealed that this was not the case ([**Sc.1**]). The regulations for the Abitur examination demanded that the candidate must be able to carry on a conversation in Latin, which was the *lingua franca* of the learned world for centuries. Since Dirichlet attended the Gymnasium just for three years, he most probably would have had problems in satisfying this crucial condition. Moreover he did not need the Abitur to study mathematics, which is what he aspired to. Nevertheless, his lacking the ability to speak Latin caused him much trouble during his career as we will see later. In any case, Dirichlet left the Gymnasium at the unusually early age of 16 years with a school-leaving certificate but without an Abitur examination.

His parents now wanted him to study law in order to secure a good living to their son. Dirichlet declared his willingness to devote himself to this bread-and-butter-education during daytime – but then he would study mathematics at night. After this his parents were wise enough to give in and gave their son their permission to study mathematics.

2. Study in Paris

Around 1820 the conditions to study mathematics in Germany were fairly bad for students really deeply interested in the subject ([**Lo**]). The only world-famous mathematician was C.F. Gauß in Göttingen, but he held a chair for astronomy and was first and foremost Director of the *Sternwarte*, and almost all his courses were devoted to astronomy, geodesy, and applied mathematics (see the list in [**Du**], p. 405 ff.). Moreover, Gauß did not like teaching – at least not on the low level which was customary at that time. On the contrary, the conditions in France were infinitely better. Eminent scientists such as P.-S. Laplace (1749–1827), A.-M. Legendre (1752–1833), J. Fourier (1768–1830), S.-D. Poisson (1781–1840), A.-L. Cauchy (1789–1857) were active in Paris, making the capital of France the world capital of mathematics. Hensel ([**H.1**], vol. 1, p. 351) informs us that Dirichlet's parents still had friendly relations with some families in Paris since the time of the French rule, and they let their son go to Paris in May 1822 to study mathematics. Dirichlet studied at the *Collège de France* and at the *Faculté des Sciences*, where he attended lectures of noted professors such as S.F. Lacroix (1765–1843), J.-B. Biot (1774–1862), J.N.P. Hachette (1769–1834), and L.B. Francœur (1773–1849). He also asked for permission to attend lectures as a guest student at the famous *École Polytechnique*. But the Prussian *chargé d'affaires* in Paris refused to ask for such a permission without the special authorization from the Prussian minister of religious, educational, and medical affairs, Karl Freiherr von Stein zum Altenstein (1770–1840). The 17-year-old student Dirichlet from a little provincial Rhenisch town had no chance to procure such an authorization.

More details about Dirichlet's courses are apparently not known. We do know that Dirichlet, besides his courses, devoted himself to a deep private study of Gauß' masterpiece *Disquisitiones arithmeticae*. At Dirichlet's request his mother had procured a copy of the *Disquisitiones* for him and sent to Paris in November 1822 (communication by G. Schubring, Bielefeld). There is no doubt that the study of Gauß' *magnum opus* left a lasting impression on Dirichlet which was of no less importance than the impression left by his courses. We know that Dirichlet studied the *Disquisitiones arithmeticae* several times during his lifetime, and we may safely assume that he was the first German mathematician who fully mastered this unique work. He never put his copy on his shelf, but always kept it on his desk. Sartorius von Waltershausen ([**Sa**], p. 21) says, that he had his copy with him on all his travels like some clergymen who always carry their prayer-book with themselves.

After one year of quiet life in seclusion devoted to his studies, Dirichlet's exterior life underwent a fundamental change in the summer of 1823. The General M.S. Foy (1775–1825) was looking for a private tutor to teach his children the German language and literature. The general was a highly cultured brilliant man and famous war hero, who held leading positions for 20 years during the wars of the French Republic and Napoléon Bonaparte. He had gained enormous popularity because of the circumspection with which he avoided unnecessary heavy losses. In 1819 Foy was elected into the Chamber of Deputies where he led the opposition and most energetically attacked the extreme royalistic and clerical policy of the majority, which voted in favour of the Bourbons. By the good offices of Larchet de Charmont, an old companion in arms of General Foy and friend of Dirichlet's parents, Dirichlet was recommended to the Foy family and got the job with a good salary, so that he no longer had to depend on his parents' financial support. The teaching duties were a modest burden, leaving Dirichlet enough time for his studies. In addition, with Dirichlet's help, Mme Foy brushed up her German, and, conversely, she helped him to get rid of his German accent when speaking French. Dirichlet was treated like a member of the Foy family and felt very much at ease in this fortunate position. The house of General Foy was a meeting-point of many celebrities of the French capital, and this enabled Dirichlet to gain self-assurance in his social bearing, which was of notable importance for his further life.

Dirichlet soon became acquainted with his academic teachers. His first work of academic character was a French translation of a paper by J.A. Eytelwein (1764–1848), member of the Royal Academy of Sciences in Berlin, on hydrodynamics ([**Ey**]). Dirichlet's teacher Hachette used this translation when he gave a report on this work to the Parisian *Société Philomatique* in May 1823, and he published a review in the *Bulletin des Sciences par la Société Philomatique de Paris*, 1823, pp. 113–115. The translation was printed in 1825 ([**Ey**]), and Dirichlet sent a copy to the Academy of Sciences in Berlin in 1826 ([**Bi.8**], p. 41).

Dirichlet's first own scientific work entitled *Mémoire sur l'impossibilité de quelques équations indéterminées du cinquième degré* ([**D.1**], pp. 1–20 and pp. 21–46) instantly gained him high scientific recognition. This work is closely related to Fermat's Last Theorem of 1637, which claims that the equation

$$x^n + y^n = z^n$$

cannot be solved in integers x, y, z all different from zero whenever $n \geq 3$ is a natural number. This topic was somehow in the air, since the French Academy of Sciences had offered a prize for a proof of this conjecture; the solution was to be submitted before January, 1818. In fact, we know that Wilhelm Olbers (1758–1840) had drawn Gauß' attention to this prize question, hoping that Gauß would be awarded the prize, a gold medal worth 3000 Francs ([**O.1**] pp. 626–627). At that time the insolubility of Fermat's equation in non-zero integers had been proved only for two exponents n, namely for $n = 4$ by Fermat himself, and for $n = 3$ by Euler. Since it suffices to prove the assertion for $n = 4$ and for all odd primes $n = p \geq 3$, the problem was open for all primes $p \geq 5$. Dirichlet attacked the case $p = 5$ and from the outset considered more generally the problem of solubility of the equation

$$x^5 \pm y^5 = Az^5$$

in integers, where A is a fixed integer. He proved for many special values of A, e.g. for $A = 4$ and for $A = 16$, that this equation admits no non-trivial solutions in integers. For the Fermat equation itself, Dirichlet showed that for any hypothetical non-trivial primitive integral solution x, y, z, one of the numbers must be divisible by 5, and he deduced a contradiction under the assumption that this number is additionally even. The "odd case" remained open at first.

Dirichlet submitted his paper to the French Academy of Sciences and got permission to lecture on his work to the members of the Academy. This must be considered a sensational event since the speaker was at that time a 20-year-old German student, who had not yet published anything and did not even have any degree. Dirichlet gave his lecture on June 11, 1825, and already one week later Lacroix and Legendre gave a very favourable report on his paper, such that the Academy decided to have it printed in the *Recueil des Mémoires des Savans étrangers*. However, the intended publication never materialized. Dirichlet himself had his work printed in 1825, and published it later on in more detailed form in the third volume of Crelle's Journal which — fortune favoured him — was founded just in time in 1826.

After that Legendre settled the aforementioned "odd case", and Dirichlet also subsequently treated this case by his methods. This solved the case $n = 5$ completely. Dirichlet had made the first significant contribution to Fermat's claim more than 50 years after Euler, and this immediately established his reputation as an excellent mathematician. Seven years later he also proved that Fermat's equation for the exponent 14 admits no non-trivial integral solution. (The case $n = 7$ was settled only in 1840 by G. Lamé (1795–1870).) A remarkable point of Dirichlet's work on Fermat's problem is that his proofs are based on considerations in quadratic fields, that is, in $\mathbb{Z}[\sqrt{5}]$ for $n = 5$, and $\mathbb{Z}[\sqrt{-7}]$ for $n = 14$. He apparently spent much more thought on the problem since he proved to be well-acquainted with the difficulties of the matter when in 1843 E. Kummer (1810–1893) gave him a manuscript containing an alleged general proof of Fermat's claim. Dirichlet returned the manuscript remarking that this would indeed be a valid proof, if Kummer had not only shown the factorization of any integer in the underlying cyclotomic field into a product of irreducible elements, but also the uniqueness of the factorization, which, however, does not hold true. Here and in Gauß' second installment on biquadratic residues we discern the beginnings of algebraic number theory.

The lecture to the Academy brought Dirichlet into closer contact with several renowned *académiciens*, notably with Fourier and Poisson, who aroused his interest in mathematical physics. The acquaintance with Fourier and the study of his *Théorie analytique de la chaleur* clearly gave him the impetus for his later epoch-making work on Fourier series (see sect. 8).

3. Entering the Prussian Civil Service

By 1807 Alexander von Humboldt (1769–1859) was living in Paris working almost single-handedly on the 36 lavishly illustrated volumes on the scientific evaluation of his 1799–1804 research expedition with A. Bonpland (1773–1858) to South and Central America. This expedition had earned him enormous world-wide fame, and he became a corresponding member of the French Academy in 1804 and a foreign member in 1810. Von Humboldt took an exceedingly broad interest in the natural sciences and beyond that, and he made generous good use of his fame to support young talents in any kind of art or science, sometimes even out of his own pocket. Around 1825 he was about to complete his great work and to return to Berlin as gentleman of the bedchamber of the Prussian King Friedrich Wilhelm III, who wanted to have such a luminary of science at his court.

On Fourier's and Poisson's recommendation Dirichlet came into contact with A. von Humboldt. For Dirichlet the search for a permanent position had become an urgent matter in 1825–1826, since General Foy died in November 1825, and the job as a private teacher would come to an end soon. J. Liouville (1809–1882) later said repeatedly that his friend Dirichlet would have stayed in Paris if it had been possible to find even a modestly paid position for him ([**T**], first part, p. 48, footnote). Even on the occasion of his first visit to A. von Humboldt, Dirichlet expressed his desire for an appointment in his homeland Prussia. Von Humboldt supported him in this plan and offered his help at once. It was his declared aim to make Berlin a centre of research in mathematics and the natural sciences ([**Bi.5**]).

With von Humboldt's help, the application to Berlin was contrived in a most promising way: On May 14, 1826, Dirichlet wrote a letter of application to the Prussian Minister von Altenstein and added a reprint of his memoir on the Fermat problem and a letter of recommendation of von Humboldt to his old friend von Altenstein. Dirichlet also sent copies of his memoir on the Fermat problem and of his translation of Eytelwein's work to the Academy in Berlin together with a letter of recommendation of A. von Humboldt, obviously hoping for support by the academicians Eytelwein and the astronomer J.F. Encke (1791–1865), a student of Gauß, and secretary to the Academy. Third, on May 28, 1826, Dirichlet sent a copy of his memoir on the Fermat problem with an accompanying letter to C.F. Gauß in Göttingen, explaining his situation and asking Gauß to submit his judgement to one of his correspondents in Berlin. Since only very few people were sufficiently acquainted with the subject of the paper, Dirichlet was concerned that his work might be underestimated in Berlin. (The letter is published in [**D.2**], p. 373–374.) He also enclosed a letter of recommendation by Gauß' acquaintance A. von Humboldt to the effect that in the opinion of Fourier and Poisson the young Dirichlet had a most brilliant talent and proceeded on the best Eulerian paths. And von Humboldt expressly asked Gauß for support of Dirichlet by means of his renown ([**Bi.6**], p. 28–29).

Now the matter proceeded smoothly: Gauß wrote to Encke that Dirichlet showed excellent talent, Encke wrote to a leading official in the ministry to the effect that, to the best of his knowledge, Gauß never had uttered such a high opinion on a scientist. After Encke had informed Gauß about the promising state of affairs, Gauß wrote on September 13, 1826, in an almost fatherly tone to Dirichlet, expressing his satisfaction to have evidence "from a letter received from the secretary of the Academy in Berlin, that we may hope that you soon will be offered an appropriate position in your homeland" ([**D.2**], pp. 375–376; [**G.1**], pp. 514–515).

Dirichlet returned to Düren in order to await the course of events. Before his return he had a meeting in Paris which might have left lasting traces in the history of mathematics. On October 24, 1826, N.H. Abel (1802–1829) wrote from Paris to his teacher and friend B.M. Holmboe (1795–1850), that he had met "Herrn Lejeune Dirichlet, a Prussian, who visited me the other day, since he considered me as a compatriot. He is a very sagacious mathematician. Simultaneously with Legendre he proved the insolubility of the equation

$$x^5 + y^5 = z^5$$

in integers and other nice things" ([**A**], French text p. 45 and Norwegian text p. 41). The meeting between Abel and Dirichlet might have been the beginning of a long friendship between fellow mathematicians, since in those days plans were being made for a polytechnic institute in Berlin, and Abel, Dirichlet, Jacobi, and the geometer J. Steiner (1796–1863) were under consideration as leading members of the staff. These plans, however, never materialized. Abel died early in 1829 just two days before Crelle sent his final message, that Abel definitely would be called to Berlin. Abel and Dirichlet never met after their brief encounter in Paris. Before that tragic end A.L. Crelle (1780–1855) had made every effort to create a position for Abel in Berlin, and he had been quite optimistic about this project until July, 1828, when he wrote to Abel the devastating news that the plan could not be carried out at that time, since a new competitor "had fallen out of the sky" ([**A**], French text, p. 66, Norwegian text, p. 55). It has been conjectured that Dirichlet was the new competitor, whose name was unknown to Abel, but recent investigations by G. Schubring (Bielefeld) show that this is not true.

In response to his application Minister von Altenstein offered Dirichlet a teaching position at the University of Breslau (Silesia, now Wrocław, Poland) with an opportunity for a *Habilitation* (qualification examination for lecturing at a university) and a modest annual salary of 400 talers, which was the usual starting salary of an associate professor at that time. (This was not too bad an offer for a 21-year-old young man without any final examination.) Von Altenstein wanted Dirichlet to move to Breslau just a few weeks later since there was a vacancy. He added, if Dirichlet had not yet passed the doctoral examination, he might send an application to the philosophical faculty of the University of Bonn which would grant him all facilities consistent with the rules ([**Sc.1**]).

The awarding of the doctorate, however, took more time than von Altenstein and Dirichlet had anticipated. The usual procedure was impossible for several formal reasons: Dirichlet had not studied at a Prussian university; his thesis, the memoir on the Fermat problem, was not written in Latin, and Dirichlet lacked experience in speaking Latin fluently and so was unable to give the required public disputation in Latin. A promotion *in absentia* was likewise impossible, since Minister von Altenstein had forbidden this kind of procedure in order to raise the level of the doctorates. To circumvent these formal problems some professors in Bonn suggested the conferment of the degree of honorary doctor. This suggestion was opposed by other members of the faculty distrustful of this way of undermining the usual rules. The discussions dragged along, but in the end the faculty voted unanimously. On February 24, 1827, Dirichlet's old friend Elvenich, at that time associate professor in Bonn, informed him about the happy ending, and a few days later Dirichlet obtained his doctor's diploma.

Because of the delay Dirichlet could not resume his teaching duties in Breslau in the winter term 1826–27. In addition, a delicate serious point still had to be settled clandstinely by the ministry. In those days Central and Eastern Europe were under the harsh rule of the Holy Alliance (1815), the Carlsbad Decrees (1819) were carried out meticulously, and alleged "demagogues" were to be prosecuted (1819). The Prussian *chargé d'affaires* in Paris received a letter from the ministry in Berlin asking if anything arousing political suspicion could be found out about the applicant, since there had been rumours that Dirichlet had lived in the house of the deceased General Foy, a fierce enemy of the government. The *chargé* checked the matter, and reported that nothing was known to the detriment of Dirichlet's views and actions, and that he apparently had lived only for his science.

4. Habilitation and Professorship in Breslau

In the course of the Prussian reforms following the Napoleonic Wars several universities were founded under the guidance of Wilhelm von Humboldt (1767–1835), Alexander von Humboldt's elder brother, namely, the Universities of Berlin (1810), Breslau (1811), and Bonn (1818), and the General Military School was founded in Berlin in 1810, on the initiative of the Prussian General G.J.D. von Scharnhorst (1755–1813). During his career Dirichlet had to do with all these institutions. We have already mentioned the honorary doctorate from Bonn.

In spring 1827, Dirichlet moved from Düren to Breslau in order to assume his appointment. On the long journey there he made a major detour via Göttingen to meet Gauß in person (March 18, 1827), and via Berlin. In a letter to his mother Dirichlet says that Gauß received him in a very friendly manner. Likewise, from a letter of Gauß to Olbers ([**O.2**], p. 479), we know that Gauß too was very much pleased to meet Dirichlet in person, and he expresses his great satisfaction that his recommendation had apparently helped Dirichlet to acquire his appointment. Gauß also tells something about the topics of the conversation, and he says that he was surprised to learn from Dirichlet, that his (i.e., Gauß') judgement on many mathematical matters completely agreed with Fourier's, notably on the foundations of geometry.

For Dirichlet, the first task in Breslau was to habilitate (qualify as a university lecturer). According to the rules in force he had
a) to give a trial lecture,
b) to write a thesis (*Habilitationsschrift*) in Latin, and
c) to defend his thesis in a public disputation to be held in Latin.
Conditions a) and b) caused no serious trouble, but Dirichlet had difficulties to meet condition c) because of his inability to speak Latin fluently. Hence he wrote to Minister von Altenstein asking for dispensation from the disputation. The minister

granted permission — very much to the displeasure of some members of the faculty ([**Bi.1**]).

To meet condition a), Dirichlet gave a trial lecture on Lambert's proof of the irrationality of the number π. In compliance with condition b), he wrote a thesis on the following number theoretic problem (see [**D.1**], pp. 45–62): Let x, b be integers, b not a square of an integer, and expand

$$(x+\sqrt{b})^n = U + V\sqrt{b} \ ,$$

where U and V are integers. The problem is to determine the linear forms containing the primes dividing V, when the variable x assumes all positive or negative integral values coprime with b. This problem is solved in two cases, viz.
(i) if n is an odd prime,
(ii) if n is a power of 2.
The results are illustrated on special examples. Of notable interest is the introduction in which Dirichlet considers examples from the theory of biquadratic residues and refers to his great work on biquadratic residues, which was to appear in Crelle's Journal at that time.

The thesis was printed early in 1828, and sent to von Altenstein, and in response Dirichlet was promoted to the rank of associate professor. A. von Humboldt added the promise to arrange Dirichlet's transfer to Berlin as soon as possible. According to Hensel ([**H.1**], vol. 1, p. 354) Dirichlet did not feel at ease in Breslau, since he did not like the widespread provincial cliquishness. Clearly, he missed the exchange of views with qualified researchers which he had enjoyed in Paris. On the other hand, there were colleagues in Breslau who held Dirichlet in high esteem, as becomes evident from a letter of Dirichlet's colleague H. Steffens (1773–1845) to the ministry ([**Bi.1**], p. 30): Steffens pointed out that Dirichlet generally was highly thought of, because of his thorough knowledge, and well liked, because of his great modesty. Moreover he wrote that his colleague — like the great Gauß in Göttingen — did not have many students, but those in the audience, who were seriously occupied with mathematics, knew how to estimate Dirichlet and how to make good use of him.

From the scientific point of view Dirichlet's time in Breslau proved to be quite successful. In April 1825, Gauß had published a first brief announcement — as he was used to doing — of his researches on biquadratic residues ([**G.1**], pp. 165–168). Recall that an integer a is called a biquadratic residue modulo the odd prime $p, p \nmid a$, if and only if the congruence $x^4 \equiv a \bmod p$ admits an integral solution. To whet his readers' appetite, Gauß communicated his results on the biquadratic character of the numbers ± 2. The full-length publication of his first installment appeared in print only in 1828 ([**G.1**], 65–92). It is well possible, though not reliably known, that Gauß talked to Dirichlet during the latter's visit to Göttingen about his recent work on biquadratic residues. In any case he did write in his very first letter of September 13, 1826, to Dirichlet about his plan to write three memoirs on this topic ([**D.2**], pp. 375–376; [**G.1**], pp. 514–515).

It is known that Gauß' announcement immediately aroused the keen interest of both Dirichlet and Jacobi, who was professor in Königsberg (East Prussia; now Kaliningrad, Russia) at that time. They both tried to find their own proofs of

Gauß' results, and they both discovered plenty of new results in the realm of higher power residues. A report on Jacobi's findings is contained in [**J.2**] amongst the correspondence with Gauß. Dirichlet discovered remarkably simple proofs of Gauß' results on the biquadratic character of ± 2, and he even answered the question as to when an odd prime q is a biquadratic residue modulo the odd prime $p, p \neq q$. To achieve the biquadratic reciprocity law, only one further step had to be taken which, however, became possible only some years later, when Gauß, in his second installment of 1832, introduced complex numbers, his Gaussian integers, into the realm of number theory ([**G.1**], pp. 169–178, 93–148, 313–385; [**R**]). This was Gauß' last long paper on number theory, and a very important one, helping to open the gate to algebraic number theory. The first printed proof of the biquadratic reciprocity law was published only in 1844 by G. Eisenstein (1823–1852; see [**Ei**], vol. 1, pp. 141–163); Jacobi had already given a proof in his lectures in Königsberg somewhat earlier.

Dirichlet succeeded with some crucial steps of his work on biquadratic residues on a brief vacation in Dresden, seven months after his visit to Gauß. Fully aware of the importance of his investigation, he immediately sent his findings in a long sealed letter to Encke in Berlin to secure his priority, and shortly thereafter he nicely described the fascinating history of his discovery in a letter of October 28, 1827, to his mother ([**R**], p. 19). In this letter he also expressed his high hopes to expect much from his new work for his further promotion and his desired transfer to Berlin. His results were published in the memoir *Recherches sur les diviseurs premiers d'une classe de formules du quatrième degré* ([**D.1**], pp. 61–98). Upon publication of this work he sent an offprint with an accompanying letter (published in [**D.2**], pp. 376–378) to Gauß, who in turn expressed his appreciation of Dirichlet's work, announced his second installment, and communicated some results carrying on the last lines of his first installment in a most surprising manner ([**D.2**], pp. 378–380; [**G.1**], pp. 516–518).

The subject of biquadratic residues was always in Dirichlet's thought up to the end of his life. In a letter of January 21, 1857, to Moritz Abraham Stern (1807–1894), Gauß' first doctoral student, who in 1859 became the first Jewish professor in Germany who did not convert to Christianity, he gave a completely elementary proof of the criterion for the biquadratic character of the number 2 ([**D.2**], p. 261 f.).

Having read Dirichlet's article, F.W. Bessel (1784–1846), the famous astronomer and colleague of Jacobi in Königsberg, enthusiastically wrote to A. von Humboldt on April 14, 1828: "... who could have imagined that this genius would succeed in reducing something appearing so difficult to such simple considerations. The name Lagrange could stand at the top of the memoir, and nobody would realize the incorrectness" ([**Bi.2**], pp. 91–92). This praise came just in time for von Humboldt to arrange Dirichlet's transfer to Berlin. Dirichlet's period of activity in Breslau was quite brief; Sturm [**St**] mentions that he lectured in Breslau only for two semesters, Kummer says three semesters.

5. Transfer to Berlin and Marriage

Aiming at Dirichlet's transfer to Berlin, A. von Humboldt sent copies of Bessel's enthusiastic letter to Minister von Altenstein and to Major J.M. von Radowitz (1797–1853), at that time teacher at the Military School in Berlin. At the same time Fourier tried to bring Dirichlet back to Paris, since he considered Dirichlet to be the right candidate to occupy a leading role in the French Academy. (It does not seem to be known, however, whether Fourier really had an offer of a definite position for Dirichlet.) Dirichlet chose Berlin, at that time a medium-sized city with 240 000 inhabitants, with dirty streets, without pavements, without street lightning, without a sewage system, without public water supply, but with many beautiful gardens.

A. von Humboldt recommended Dirichlet to Major von Radowitz and to the minister of war for a vacant post at the Military School. At first there were some reservations to installing a young man just 23 years of age for the instruction of officers. Hence Dirichlet was first employed on probation only. At the same time he was granted leave for one year from his duties in Breslau. During this time his salary was paid further on from Breslau; in addition he received 600 talers per year from the Military School. The trial period was successful, and the leave from Breslau was extended twice, so that he never went back there.

From the very beginning, Dirichlet also had applied for permission to give lectures at the University of Berlin, and in 1831 he was formally transferred to the philosophical faculty of the University of Berlin with the further duty to teach at the Military School. There were, however, strange formal oddities about his legal status at the University of Berlin which will be dealt with in sect. 7.

In the same year 1831 he was elected to the Royal Academy of Sciences in Berlin, and upon confirmation by the king, the election became effective in 1832. At that time the 27-year-old Dirichlet was the youngest member of the Academy.

Shortly after Dirichlet's move to Berlin, a most prestigious scientific event organized by A. von Humboldt was held there, the seventh assembly of the German Association of Scientists and Physicians (September 18–26, 1828). More than 600 participants from Germany and abroad attended the meeting, Felix Mendelssohn Bartholdy composed a ceremonial music, the poet Rellstab wrote a special poem, a stage design by Schinkel for the aria of the Queen of the Night in Mozart's *Magic Flute* was used for decoration, with the names of famous scientists written in the firmament. A great gala dinner for all participants and special invited guests with the king attending was held at von Humboldt's expense. Gauß took part in the meeting and lived as a special guest in von Humboldt's house. Dirichlet was invited by von Humboldt jointly with Gauß, Charles Babbage (1792–1871) and the officers von Radowitz and K. von Müffing (1775–1851) as a step towards employment at the Military School. Another participant of the conference was the young physicist Wilhelm Weber (1804–1891), at that time associate professor at the University of Halle. Gauß got to know Weber at this assembly, and in 1831 he arranged Weber's call to Göttingen, where they both started their famous joint work on the investigation of electromagnetism. The stimulating atmosphere in Berlin was compared by Gauß in a letter to his former student C.L. Gerling (1788–1864) in Marburg "to a move from atmospheric air to oxygen".

The following years were the happiest in Dirichlet's life both from the professional and the private point of view. Once more it was A. von Humboldt who established also the private relationship. At that time great salons were held in Berlin, where people active in art, science, humanities, politics, military affairs, economics, etc. met regularly, say, once per week. A. von Humboldt introduced Dirichlet to the house of Abraham Mendelssohn Bartholdy (1776–1835) (son of the legendary Moses Mendelssohn (1729–1786)) and his wife Lea, née Salomon (1777–1842), which was a unique meeting point of the cultured Berlin. The Mendelssohn family lived in a baroque palace erected in 1735, with a two-storied main building, side-wings, a large garden hall holding up to 300 persons, and a huge garden of approximately 3 hectares (almost 10 acres) size. (The premises were sold in 1851 to the Prussian state and the house became the seat of the Upper Chamber of the Prussian Parliament. In 1904 a new building was erected, which successively housed the Upper Chamber of the Prussian Parliament, the Prussian Council of State, the Cabinet of the GDR, and presently the German Bundesrat.) There is much to be told about the Mendelssohn family which has to be omitted here; for more information see the recent wonderful book by T. Lackmann [**Lac**]. Every Sunday morning famous Sunday concerts were given in the Mendelssohn garden hall with the four highly gifted Mendelssohn children performing. These were the pianist and composer Fanny (1805–1847), later married to the painter Wilhelm Hensel (1794–1861), the musical prodigy, brilliant pianist and composer Felix (1809–1847), the musically gifted Rebecka (1811–1858), and the cellist Paul (1812–1874), who later carried out the family's banking operations. Sunday concerts started at 11 o'clock and lasted for 4 hours with a break for conversation and refreshments in between. Wilhelm Hensel made portraits of the guests — more than 1000 portraits came into being this way, a unique document of the cultural history of that time.

From the very beginning, Dirichlet took an interest in Rebecka, and although she had many suitors, she decided for Dirichlet. Lackmann ([**Lac**]) characterizes Rebecka as the linguistically most gifted, wittiest, and politically most receptive of the four children. She experienced the radical changes during the first half of the nineteeth century more consciously and critically than her siblings. These traits are clearly discernible also from her letters quoted by her nephew Sebastian Hensel ([**H.1**], [**H.2**]). The engagement to Dirichlet took place in November 1831. After the wedding in May 1832, the young married couple moved into a flat in the parental house, Leipziger Str. 3, and after the Italian journey (1843–1845), the Dirichlet family moved to Leipziger Platz 18.

In 1832 Dirichlet's life could have taken quite a different course. Gauß planned to nominate Dirichlet as a successor to his deceased colleague, the mathematician B.F. Thibaut (1775–1832). When Gauß learnt about Dirichlet's marriage, he cancelled this plan, since he assumed that Dirichlet would not be willing to leave Berlin. The triumvirate Gauß, Dirichlet, and Weber would have given Göttingen a unique constellation in mathematics and natural sciences not to be found anywhere else in the world.

Dirichlet was notoriously lazy about letter writing. He obviously preferred to settle matters by directly contacting people. On July 2, 1833, the first child, the son Walter, was born to the Dirichlet family. Grandfather Abraham Mendelssohn Bartholdy got the happy news on a buisiness trip in London. In a letter he congratulated Rebecka and continued resentfully: "I don't congratulate Dirichlet, at least not in writing, since he had the heart not to write me a single word, even on this occasion; at least he could have written: $2 + 1 = 3$" ([**H.1**], vol. 1, pp. 340–341). (Walter Dirichlet became a well-known politician later and member of the German Reichstag 1881–1887; see [**Ah.1**], 2. Teil, p. 51.)

The Mendelssohn family is closely related with many artists and scientists of whom we but mention some prominent mathematicians: The renowned number theorist Ernst Eduard Kummer was married to Rebecka's cousin Ottilie Mendelssohn (1819–1848) and hence was Dirichlet's cousin. He later became Dirichlet's successor at the University of Berlin and at the Military School, when Dirichlet left for Göttingen. The function theorist Hermann Amandus Schwarz (1843–1921), after whom Schwarz' Lemma and the Cauchy–Schwarz Inequality are named, was married to Kummer's daughter Marie Elisabeth, and hence was Kummer's son-in-law. The analyst Heinrich Eduard Heine (1821–1881), after whom the Heine–Borel Theorem got its name, was a brother of Albertine Mendelssohn Bartholdy, née Heine, wife of Rebecka's brother Paul. Kurt Hensel (1861–1941), discoverer of the p-adic numbers and for many years editor of Crelle's Journal, was a son of Sebastian Hensel (1830–1898) and his wife Julie, née Adelson; Sebastian Hensel was the only child of Fanny and Wilhelm Hensel, and hence a nephew of the Dirichlets. Kurt and Gertrud (née Hahn) Hensel's daughter Ruth Therese was married to the professor of law Franz Haymann, and the noted function theorist Walter Hayman (born 1926) is an offspring of this married couple. The noted group theorist and number theorist Robert Remak (1888– some unknown day after 1942 when he met his death in Auschwitz) was a nephew of Kurt and Gertrud Hensel. The philosopher and logician Leonard Nelson (1882–1927) was a great-great-grandson of Gustav and Rebecka Lejeune Dirichlet.

6. Teaching at the Military School

When Dirichlet began teaching at the Military School on October 1, 1828, he first worked as a coach for the course of F.T. Poselger (1771–1838). It is a curious coincidence that Georg Simon Ohm, Dirichlet's mathematics teacher at the *Gymnasium* in Cologne, simultaneously also worked as a coach for the course of his brother, the mathematician Martin Ohm (1792–1872), who was professor at the University of Berlin. Dirichlet's regular teaching started one year later, on October 1, 1829. The course went on for three years and then started anew. Its content was essentially elementary and practical in nature, starting in the first year with the theory of equations (up to polynomial equations of the fourth degree), elementary theory of series, some stereometry and descriptive geometry. This was followed in the second year by some trigonometry, the theory of conics, more stereometry and analytical geometry of three-dimensional space. The third year was devoted to mechanics, hydromechanics, mathematical geography and geodesy. At first, the differential and integral calculus was not included in the curriculum, but some years later Dirichlet succeeded in raising the level of instruction by introducing so-called higher analysis and its applications to problems of mechanics into the program. Subsequently, this change became permanent and was adhered to even when Dirichlet left his post ([**Lam**]). Altogether he taught for 27 years at the Military School, from his transfer to Berlin in 1828 to his move to Göttingen in 1855, with two breaks during his Italian journey (1843–1845) and after the March Revolution of 1848 in Berlin, when the Military School was closed down for some time, causing Dirichlet a sizable loss of his income.

During the first years Dirichlet really enjoyed his position at the Military School. He proved to be an excellent teacher, whose courses were very much appreciated by his audience, and he liked consorting with the young officers, who were almost of his own age. His refined manners impressed the officers, and he invited them for stimulating evening parties in the course of which he usually formed the centre of conversation. Over the years, however, he got tired of repeating the same curriculum every three years. Moreover, he urgently needed more time for his research; together with his lectures at the university his teaching load typically was around 18 hours per week.

When the Military School was reopened after the 1848 revolution, a new reactionary spirit had emerged among the officers, who as a rule belonged to the nobility. This was quite opposed to Dirichlet's own very liberal convictions. His desire to quit the post at the Military School grew, but he needed a compensation for his loss in income from that position, since his payment at the University of Berlin was rather modest. When the Prussian ministry was overly reluctant to comply with his wishes, he accepted the most prestigious call to Göttingen as a successor to Gauß in 1855.

7. Dirichlet as a Professor at the University of Berlin

From the very beginning Dirichlet applied for permission to give lectures at the University of Berlin. The minister approved his application and communicated this decision to the philosophical faculty. But the faculty protested, since Dirichlet was neither habilitated nor appointed professor, whence the instruction of the minister was against the rules. In his response the minister showed himself conciliatory and said he would leave it to the faculty to demand from Dirichlet an appropriate achievement for his *Habilitation*. Thereupon the dean of the philosphical faculty offered a reasonable solution: He suggested that the faculty would consider Dirichlet — in view of his merits — as *Professor designatus*, with the right to give lectures. To satisfy the formalities of a *Habilitation*, he only requested Dirichlet

a) to distribute a written program in Latin, and
b) to give a lecture in Latin in the large lecture-hall.

This seemed to be a generous solution. Dirichlet was well able to compose texts in Latin as he had proved in Breslau with his *Habilitationsschrift*. He could prepare his lecture in writing and just read it — this did not seem to take great pains. But quite unexpectedly he gave the lecture only with enormous reluctance. It took Dirichlet almost 23 years to give it. The lecture was entitled *De formarum* *binarium secundi gradus compositione* ("On the composition of binary quadratic forms"; [**D.2**], pp. 105–114) and comprises less than 8 printed pages. On the title page Dirichlet is referred to as *Phil. Doct. Prof. Publ. Ord. Design.* The reasons for the unbelievable delay are given in a letter to the dean H.W. Dove (1803–1879) of November 10, 1850, quoted in [**Bi.1**], p. 43. In the meantime Dirichlet was transferred for long as an associate professor to the University of Berlin in 1831, and he was even advanced to the rank of full professor in 1839, but in the faculty he still remained *Professor designatus* up to his *Habilitation* in 1851. This meant that it was only in 1851 that he had equal rights in the faculty; before that time he was, e.g. not entitled to write reports on doctoral dissertations nor could he influence *Habilitationen* — obviously a strange situation since Dirichlet was by far the most competent mathematician on the faculty.

We have several reports of eye-witnesses about Dirichlet's lectures and his social life. After his participation in the assembly of the German Association of Scientists and Physicians, Wilhelm Weber started a research stay in Berlin beginning in October, 1828. Following the advice of A. von Humboldt, he attended Dirichlet's lectures on Fourier's theory of heat. The eager student became an intimate friend of Dirichlet's, who later played a vital role in the negotiations leading to Dirichlet's move to Göttingen (see sect. 12). We quote some lines of the physicist Heinrich Weber (1839–1928), nephew of Wilhelm Weber, not to be confused with the mathematician Heinrich Weber (1842–1913), which give some impression on the social life of his uncle in Berlin ([**Web**], pp. 14–15): "After the lectures which were given three times per week from 12 to 1 o'clock there used to be a walk in which Dirichlet often took part, and in the afternoon it became eventually common practice to go to the coffee-house 'Dirichlet'. 'After the lecture every time one of us invites the others without further ado to have coffee at Dirichlet's, where we show up at 2 or 3 o'clock and stay quite cheerfully up to 6 o'clock'[3]".

During his first years in Berlin Dirichlet had only rather few students, numbers varying typically between 5 and 10. Some lectures could not even be given at all for lack of students. This is not surprising since Dirichlet generally gave lectures on what were considered to be "higher" topics, whereas the great majority of the students preferred the lectures of Dirichlet's colleagues, which were not so demanding and more oriented towards the final examination. Before long, however, the situation changed, Dirichlet's reputation as an excellent teacher became generally known, and audiences comprised typically between 20 and 40 students, which was quite a large audience at that time.

Although Dirichlet was not on the face of it a brilliant speaker like Jacobi, the great clarity of his thought, his striving for perfection, the self-confidence with which he elaborated on the most complicated matters, and his thoughtful remarks fascinated his students. Whereas mere computations played a major role in the lectures of most of his contemporaries, in Dirichlet's lectures the mathematical argument came to the fore. In this regard Minkowski [**Mi**] speaks "of the other Dirichlet Principle to overcome the problems with a minimum of blind computation and a maximum of penetrating thought", and from that time on he dates "the modern times in the history of mathematics".

[3] Quotation from a family letter of W. Weber of November 21, 1828.

Dirichlet prepared his lectures carefully and spoke without notes. When he could not finish a longer development, he jotted down the last formula on a slip of paper, which he drew out of his pocket at the beginning of the next lecture to continue the argument. A vivid description of his lecturing habits was given by Karl Emil Gruhl (1833–1917), who attended his lectures in Berlin (1853–1855) and who later became a leading official in the Prussian ministry of education (see [**Sc.2**]). An admiring description of Dirichlet's teaching has been passed on to us by Thomas Archer Hirst (1830–1892), who was awarded a doctor's degree in Marburg, Germany, in 1852, and after that studied with Dirichlet and Steiner in Berlin. In Hirst's diary we find the following entry of October 31, 1852 ([**GW**], p. 623): "Dirichlet cannot be surpassed for richness of material and clear insight into it: as a speaker he has no advantages — there is nothing like fluency about him, and yet a clear eye and understanding make it dispensable: without an effort you would not notice his hesitating speech. What is peculiar in him, he never sees his audience — when he does not use the blackboard at which time his back is turned to us, he sits at the high desk facing us, puts his spectacles up on his forehead, leans his head on both hands, and keeps his eyes, when not covered with his hands, mostly shut. He uses no notes, inside his hands he sees an imaginary calculation, and reads it out to us — that we understand it as well as if we too saw it. I like that kind of lecturing." — After Hirst called on Dirichlet and was "met with a very hearty reception", he noted in his diary on October 13, 1852 ([**GW**], p. 622): "He is a rather tall, lanky-looking man, with moustache and beard about to turn grey (perhaps 45 years old), with a somewhat harsh voice and rather deaf: it was early, he was unwashed, and unshaved (what of him required shaving), with his 'Schlafrock', slippers, cup of coffee and cigar ... I thought, as we sat each at an end of the sofa, and the smoke of our cigars carried question and answer to and fro, and intermingled in graceful curves before it rose to the ceiling and mixed with the common atmospheric air, 'If all be well, we will smoke our friendly cigar together many a time yet, good-natured Lejeune Dirichlet'."

The topics of Dirichlet's lectures were mainly chosen from various areas of number theory, foundations of analysis (including infinite series, applications of integral calculus), and mathematical physics. He was the first university teacher in Germany to give lectures on his favourite subject, number theory, and on the application of analytical techniques to number theory; 23 of his lectures were devoted to these topics ([**Bi.1**]; [**Bi.8**], p. 47).

Most importantly, the lectures of Jacobi in Königsberg and Dirichlet in Berlin gave the impetus for a general rise of the level of mathematical instruction in Germany, which ultimately led to the very high standards of university mathematics in Germany in the second half of the nineteenth century and beyond that up to 1933. Jacobi even established a kind of "Königsberg school" of mathematics principally dedicated to the investigation of the theory of elliptic functions. The foundation of the first mathematical seminar in Germany in Königsberg (1834) was an important event in his teaching activities. Dirichlet was less extroverted; from 1834 onwards he conducted a kind of private mathematical seminar in his house which was not even mentioned in the university calendar. The aim of this private seminar was to give his students an opportunity to practice their oral presentation and their skill in solving problems. For a full-length account on the development of the study of mathematics at German universities during the nineteenth century see Lorey [**Lo**].

A large number of mathematicians received formative impressions from Dirichlet by his lectures or by personal contacts. Without striving for a complete list we mention the names of P. Bachmann (1837–1920), the author of numerous books on number theory, G. Bauer (1820–1907), professor in Munich, C.W. Borchardt (1817–1880), Crelle's successor as editor of Crelle's Journal, M. Cantor (1829–1920), a leading German historian of mathematics of his time, E.B. Christoffel (1829–1900), known for his work on differential geometry, R. Dedekind (1831–1916), noted for his truly fundamental work on algebra and algebraic number theory, G. Eisenstein (1823–1852), noted for his profound work on number theory and elliptic functions, A. Enneper (1830–1885), known for his work on the theory of surfaces and elliptic functions, E. Heine (1821–1881), after whom the Heine–Borel Theorem got its name, L. Kronecker (1823–1891), the editor of Dirichlet's collected works, who jointly with Kummer and Weierstraß made Berlin a world centre of mathematics in the second half of the nineteenth century, E.E. Kummer (1810–1893), one of the most important number theorists of the nineteenth century and not only Dirichlet's successor in his chair in Berlin but also the author of the important obituary [**Ku**] on Dirichlet, R. Lipschitz (1832–1903), noted for his work on analysis and number theory, B. Riemann (1826–1866), one of the greatest mathematicians of the 19th century and Dirichlet's successor in Göttingen, E. Schering (1833–1897), editor of the first edition of the first 6 volumes of Gauß' collected works, H. Schröter (1829–1892), professor in Breslau, L. von Seidel (1821–1896), professor in Munich, who introduced the notion of uniform convergence, J. Weingarten (1836–1910), who advanced the theory of surfaces.

Dirichlet's lectures had a lasting effect even beyond the grave, although he did not prepare notes. After his death several of his former students published books based on his lectures: In 1904 G. Arendt (1832–1915) edited Dirichlet's lectures on definite integrals following his 1854 Berlin lectures ([**D.7**]). As early as 1871 G.F. Meyer (1834–1905) had published the 1858 Göttingen lectures on the same topic ([**MG**]), but his account does not follow Dirichlet's lectures as closely as Arendt does. The lectures on "forces inversely proportional to the square of the distance" were published by F. Grube (1835–1893) in 1876 ([**Gr**]). Here one may read how Dirichlet himself explained what Riemann later called "Dirichlet's Principle". And last but not least, there are Dirichlet's lectures on number theory in the masterly edition of R. Dedekind, who over the years enlarged his own additions to a pioneering exposition of the foundations of algebraic number theory based on the concept of ideal.

8. Mathematical Works

In spite of his heavy teaching load, Dirichlet achieved research results of the highest quality during his years in Berlin. When A. von Humboldt asked Gauß in 1845 for a proposal of a candidate for the order *pour le mérite*, Gauß did "not neglect to nominate Professor Dirichlet in Berlin. The same has — as far as I know — not yet published a big work, and also his individual memoirs do not yet comprise a big volume. But they are jewels, and one does not weigh jewels on a grocer's scales"

([**Bi.6**], p. 88)[4]. We quote a few highlights of Dirichlet's *œuvre* showing him at the peak of his creative power.

A. Fourier Series. The question whether or not an "arbitrary" 2π-periodic function on the real line can be expanded into a trigonometric series

$$\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

was the subject of controversal discussions among the great analysts of the eighteenth century, such as L. Euler, J. d'Alembert, D. Bernoulli, J. Lagrange. Fourier himself did not settle this problem, though he and his predecessors knew that such an expansion exists in many interesting cases. Dirichlet was the first mathematician to prove rigorously for a fairly wide class of functions that such an expansion is possible. His justly famous memoir on this topic is entitled *Sur la convergence des séries trigonométriques qui servent à représenter une fonction arbitraire entre des limites données* (1829) ([**D.1**], pp. 117–132). He points out in this work that some restriction on the behaviour of the function in question is necessary for a positive solution to the problem, since, e.g. the notion of integral *"ne signifie quelque chose"* for the (Dirichlet) function

$$f(x) = \begin{cases} c & \text{for} \quad x \in \mathbb{Q}\,, \\ d & \text{for} \quad x \in \mathbb{R} \setminus \mathbb{Q}\,, \end{cases}$$

whenever $c, d \in \mathbb{R}, c \neq d$ ([**D.1**], p. 132). An extended version of his work appeared in 1837 in German ([**D.1**], pp. 133–160; [**D.4**]). We comment on this German version since it contains various issues of general interest. Before dealing with his main problem, Dirichlet clarifies some points which nowadays belong to any introductory course on real analysis, but which were by far not equally commonplace at that time. This refers first of all to the notion of function. In Euler's *Introductio in analysin infinitorum* the notion of function is circumscribed somewhat tentatively by means of "analytical expressions", but in his book on differential calculus his notion of function is so wide "as to comprise all manners by which one magnitude may be determined by another one". This very wide concept, however, was not generally accepted. But then Fourier in his *Théorie analytique de la chaleur* (1822) advanced the opinion that also any non-connected curve may be represented by a trigonometric series, and he formulated a corresponding general notion of function. Dirichlet follows Fourier in his 1837 memoir: "If to any x there corresponds a single finite y, namely in such a way that, when x continuously runs through the interval from a to b, $y = f(x)$ likewise varies little by little, then y is called a continuous ... function of x. Yet it is not necessary that y in this whole interval depend on x according to the same law; one need not even think of a dependence expressible in terms of mathematical operations" ([**D.1**], p. 135). This definition suffices for Dirichlet since he only considers piecewise continuous functions.

Then Dirichlet defines the integral for a continuous function on $[a, b]$ as the limit of decomposition sums for equidistant decompositions, when the number of intermediate points tends to infinity. Since his paper is written for a manual of physics, he does not formally prove the existence of this limit, but in his lectures [**D.7**] he fully

[4]At that time Dirichlet was not yet awarded the order. He got it in 1855 after Gauß' death, and thus became successor to Gauß also as a recipient of this extraordinary honour.

proves the existence by means of the uniform continuity of a continuous function on a closed interval, which he calls a "fundamental property of continuous functions" (loc. cit., p. 7).

He then tentatively approaches the development into a trigonometric series by means of discretization. This makes the final result plausible, but leaves the crucial limit process unproved. Hence he starts anew in the same way customary today: Given the piecewise continuous[5] 2π-periodic function $f : \mathbb{R} \to \mathbb{R}$, he forms the (Euler-)Fourier coefficients

$$\begin{aligned} a_k &:= \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\,\cos kt\,dt \quad (k \geq 0)\ , \\ b_k &:= \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\,\sin kt\,dt \quad (k \geq 1)\ , \end{aligned}$$

and transforms the partial sum

$$s_n(x) := \frac{a_0}{2} + \sum_{k=1}^{n}(a_k \cos kx + b_k \sin kx)$$

($n \geq 0$) into an integral, nowadays known as *Dirichlet's Integral*,

$$s_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\frac{\sin(2n+1)\frac{t-x}{2}}{\sin\frac{t-x}{2}}\,dt\ .$$

The pioneering progress of Dirichlet's work now is to find a precise simple sufficient condition implying

$$\lim_{n\to\infty} s_n(x) = \frac{1}{2}(f(x+0) + f(x-0))\ ,$$

namely, this limit relation holds whenever f is piecewise continuous and piecewise monotone in a neighbourhood of x. A crucial role in Dirichlet's argument is played by a preliminary version of what is now known as the Riemann–Lebesgue Lemma and by a mean-value theorem for integrals.

Using the same method Dirichlet also proves the expansion of an "arbitrary" function depending on two angles into a series of spherical functions ([**D.1**], pp. 283–306). The main trick of this paper is a transformation of the partial sum into an integral of the shape of Dirichlet's Integral.

A characteristic feature of Dirichlet's work is his skilful application of analysis to questions of number theory, which made him the founder of analytic number theory ([**Sh**]). This trait of his work appears for the first time in his paper *Über eine neue Anwendung bestimmter Integrale auf die Summation endlicher oder unendlicher Reihen* (1835) (On a new application of definite integrals to the summation of finite or infinite series, [**D.1**], pp. 237–256; shortened French translation in [**D.1**], pp. 257–270). Applying his result on the limiting behaviour of Dirichlet's Integral for n tending to infinity, he computes the Gaussian Sums in a most lucid way, and he uses the latter result to give an ingenious proof of the quadratic reciprocity theorem. (Recall that Gauß himself published 6 different proofs of his *theorema fundamentale*, the law of quadratic reciprocity (see [**G.2**]).)

[5] finitely many pieces in $[0, 2\pi]$

B. Dirichlet's Theorem on Primes in Arithmetical Progressions. Dirichlet's mastery in the application of analysis to number theory manifests itself most impressively in his proof of the theorem on an infinitude of primes in any arithmetic progression of the form $(a+km)_{k\geq 1}$, where a and m are coprime natural numbers. In order to explain why this theorem is of special interest, Dirichlet gives the following typical example ([**D.1**], p. 309): The law of quadratic reciprocity implies that the congruence $x^2+7 \equiv 0\,(\bmod\ p)$ is solvable precisely for those primes p different from 2 and 7 which are of the form $7k+1, 7k+2$, or $7k+4$ for some natural number k. But the law of quadratic reciprocity gives no information at all about the existence of primes in any of these arithmetic progressions.

Dirichlet's theorem on primes in arithmetic progressions was first published in German in 1837 (see [**D.1**], pp. 307–312 and pp. 313–342); a French translation was published in *Liouville's Journal*, but not included in Dirichlet's collected papers (see [**D.2**], p. 421). In this work, Dirichlet again utilizes the opportunity to clarify some points of general interest which were not commonplace at that time. Prior to his introduction of the L-series he explains the "essential difference" which "exists between two kinds of infinite series. If one considers instead of each term its absolute value, or, if it is complex, its modulus, two cases may occur. Either one may find a finite magnitude exceeding any finite sum of arbitrarily many of these absolute values or moduli, or this condition is not satisfied by any finite number however large. In the first case, the series always converges and has a unique definite sum irrespective of the order of the terms, no matter if these proceed in one dimension or if they proceed in two or more dimensions forming a so-called double series or multiple series. In the second case, the series may still be convergent, but this property as well as the sum will depend in an essential way on the order of the terms. Whenever convergence occurs for a certain order it may fail for another order, or, if this is not the case, the sum of the series may be quite a different one" ([**D.1**], p. 318).

The crucial new tools enabling Dirichlet to prove his theorem are the L-series which nowadays bear his name. In the original work these series were introduced by means of suitable primitive roots and roots of unity, which are the values of the characters. This makes the representation somewhat lengthy and technical (see e.g. [**Lan**], vol. I, p. 391 ff. or [**N.2**], p. 51 ff.). For the sake of conciseness we use the modern language of characters: By definition, a Dirichlet character $\bmod m$ is a homomorphism $\chi : (\mathbb{Z}/m\mathbb{Z})^\times \to S^1$, where $(\mathbb{Z}/m\mathbb{Z})^\times$ denotes the group of prime residue classes $\bmod m$ and S^1 the unit circle in $\mathbb{C}$. To any such χ corresponds a map (by abuse of notation likewise denoted by the same letter) $\chi : \mathbb{Z} \to \mathbb{C}$ such that
a) $\chi(n) = 0$ if and only if $(m, n) > 1$,
b) $\chi(kn) = \chi(k)\chi(n)$ for all $k, n \in \mathbb{Z}$,
c) $\chi(n) = \chi(k)$ whenever $k \equiv n \,(\bmod\, m)$,
namely, $\chi(n) := \chi(n + m\mathbb{Z})$ if $(m, n) = 1$.
The set of Dirichlet characters $\bmod m$ is a multiplicative group isomorphic to $(\mathbb{Z}/m\mathbb{Z})^\times$ with the so-called principal character χ_0 as neutral element. To any such χ Dirichlet associates an L-series

$$L(s,\chi) := \sum_{n=1}^{\infty} \frac{\chi(n)}{n^s} \quad (s > 1)\ ,$$

and expands it into an Euler product

$$L(s,\chi) = \prod_p (1-\chi(p)p^{-s})^{-1} \ ,$$

where the product extends over all primes p. He then defines the logarithm

$$\log L(s,\chi) = \sum_p \sum_{k=1}^{\infty} \frac{1}{k} \frac{\chi(p)^k}{p^{ks}} \quad (s>1)$$

and uses it to sift the primes in the progression $(a+km)_{k\geq 1}$ by means of a summation over all $\phi(m)$ Dirichlet characters χ mod m:

$$\begin{aligned} \frac{1}{\phi(m)} \sum_\chi \overline{\chi(a)} \log L(s,\chi) &= \sum_{\substack{k\geq 1, p \\ p^k \equiv a \,(\mathrm{mod}\, m)}} \frac{1}{kp^{ks}} \\ &= \sum_{p\equiv a \,(\mathrm{mod}\, m)} \frac{1}{p^s} + R(s) \ . \end{aligned} \tag{1}$$

Here, $R(s)$ is the contribution of the terms with $k \geq 2$ which converges absolutely for $s > \frac{1}{2}$. For $\chi \neq \chi_0$ the series $L(s,\chi)$ even converges for $s > 0$ and is continuous in s. Dirichlet's great discovery now is that

$$L(1,\chi) \neq 0 \quad \text{for } \chi \neq \chi_0 \ . \tag{2}$$

Combining this with the simple observation that $L(s,\chi_0) \to \infty$ as $s \to 1+0$, formula (1) yields

$$\sum_{p\equiv a \text{ mod } m} \frac{1}{p^s} \longrightarrow \infty \quad \text{for } s \to 1+0$$

which gives the desired result. To be precise, in his 1837 paper Dirichlet proved (2) only for prime numbers m, but he pointed out that in the original draft of his paper he also proved (2) for arbitrary natural numbers m by means of "indirect and rather complicated considerations. Later I convinced myself that the same aim may be achieved by a different method in a much shorter way" ([**D.1**], p. 342). By this he means his class number formula which makes the non-vanishing of $L(1,\chi)$ obvious (see section C).

Dirichlet's theorem on primes in arithmetic progressions holds analogously for $\mathbb{Z}[i]$ instead of $\mathbb{Z}$. This was shown by Dirichlet himself in another paper in 1841 ([**D.1**], pp. 503–508 and pp. 509–532).

C. Dirichlet's Class Number Formula. On September 10, 1838, C.G.J. Jacobi wrote to his brother Moritz Hermann Jacobi (1801–1874), a renowned physicist in St. Petersburg, with unreserved admiration: "Applying Fourier series to number theory, Dirichlet has recently found results touching the utmost of human acumen" ([**Ah.2**], p. 47). This remark goes back to a letter of Dirichlet's to Jacobi on his research on the determination of the class number of binary quadratic forms with fixed determinant. Dirichlet first sketched his results on this topic and on the mean value of certain arithmetic functions in 1838 in an article in Crelle's Journal ([**D.1**], pp. 357–374) and elaborated on the matter in full detail in a very long memoir of 1839–1840, likewise in Crelle's Journal ([**D.1**], pp. 411–496; [**D.3**]).

Following Gauß, Dirichlet considered quadratic forms

$$ax^2 + 2bxy + cy^2$$

with even middle coefficient $2b$. This entails a large number of cases such that the class number formula finally appears in 8 different versions, 4 for positive and 4 for negative determinants. Later on Kronecker found out that the matter can be dealt with much more concisely if one considers from the very beginning forms of the shape

$$f(x, y) := ax^2 + bxy + cy^2 \ . \tag{3}$$

He published only a brief outline of the necessary modifications in the framework of his investigations on elliptic functions ([**Kr**], pp. 371–375); an exposition of book-length was subsequently given by de Seguier ([**Se**]).

For simplicity, we follow Kronecker's approach and consider quadratic forms of the type (3) with integral coefficients a, b, c and discriminant $D = b^2 - 4ac$ assuming that D is not the square of an integer. The crucial question is whether or not an integer n can be represented by the form (3) by attributing suitable integral values to x, y. This question admits no simple answer as long as we consider an individual form f.

The substitution

$$\begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \text{ with } \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \in \mathrm{SL}_2(\mathbb{Z})$$

transforms f into a so-called *(properly) equivalent* form

$$f'(x, y) = a'x^2 + b'xy + c'y^2$$

which evidently has the same discriminant and represents the same integers. Hence the problem of representation needs to be solved only for a representative system of the *finitely many* equivalence classes of binary forms of fixed discriminant D. Associated with each form f is its group of *automorphs* containing all matrices $\left(\begin{smallmatrix} \alpha & \beta \\ \gamma & \delta \end{smallmatrix}\right) \in \mathrm{SL}_2(\mathbb{Z})$ transforming f into itself. The really interesting quantity now is the number $R(n, f)$ of representations of n by f which are inequivalent with respect to the natural action of the group of automorphs. Then $R(n, f)$ turns out to be finite, but still there is no simple formula for this quantity.

Define now f to be *primitive* if $(a, b, c) = 1$. Forms equivalent to primitive ones are primitive. Denote by $f_1, \dots, f_h$ a representative system of primitive binary quadratic forms of discriminant D, where $h = h(D)$ is called the *class number*. For $D < 0$ we tacitly assume that $f_1, \dots, f_h$ are positive definite. Moreover we assume that D is a *fundamental discriminant*, that is, D is an integer satisfying either

(i) $D \equiv 1 \pmod 4, D$ square-free, or
(ii) $D \equiv 0 \pmod 4, \frac{D}{4} \equiv 2$ or $3 \pmod 4, \frac{D}{4}$ square-free.

Then there is the simple formula

$$\sum_{j=1}^{h} R(n, f_j) = \sum_{m \mid n} \left(\frac{D}{m}\right) \quad (n \neq 0) \ ,$$

where $\left(\frac{D}{\cdot}\right)$ is the so-called *Kronecker symbol*, an extension of the familiar Legendre symbol ([**Z**], p. 38). The law of quadratic reciprocity implies that $n \mapsto \left(\frac{D}{n}\right)$ is a so-called primitive Dirichlet character $\bmod |D|$. It is known that any primitive real Dirichlet character is one of the characters $\left(\frac{D}{\cdot}\right)$ for some fundamental discriminant D. In terms of generating functions the last sum formula means, supposing that $D < 0$,

$$\sum_{j=1}^{h} \sum_{(x,y)\neq(0,0)} (f_j(x,y))^{-s} = w\zeta(s) L\left(s, \left(\frac{D}{\cdot}\right)\right)$$

with $w = 2, 4$ or 6 as $D < -4, D = -4$ or $D = -3$, respectively. Using geometric considerations, Dirichlet deduces by a limiting process the first of his class number formulae

$$h(D) = \begin{cases} \dfrac{w\sqrt{|D|}}{2\pi} L\left(1, \left(\dfrac{D}{\cdot}\right)\right) & \text{if} \quad D < 0\,, \\ \dfrac{\sqrt{D}}{\log \varepsilon_0} L\left(1, \left(\dfrac{D}{\cdot}\right)\right) & \text{if} \quad D > 0\,. \end{cases} \tag{4}$$

In the second formula, $\varepsilon_0 = \frac{1}{2}(t_0 + u_0\sqrt{D})$ denotes the fundamental solution of Pell's equation $t^2 - Du^2 = 4$ (with $t_0, u_0 > 0$ minimal). The case $D > 0$ is decidedly more difficult than the case $D < 0$ because of the more difficult description of the (infinite) group of automorphs in terms of the solutions of Pell's equation. Formula (4) continues to hold even if D is a general discriminant ([**Z**], p. 73 f.). The class number being positive and finite, Dirichlet was able to conclude the non-vanishing of $L(1, \chi)$ (in the crucial case of a real character) mentioned above.

Using Gauß sums Dirichlet was moreover able to compute the values of the L-series in (4) in a simple closed form. This yields

$$h(D) = \begin{cases} -\dfrac{w}{2|D|} \displaystyle\sum_{n=1}^{|D|-1} \left(\dfrac{D}{n}\right) n & \text{for} \quad D < 0\,, \\ -\dfrac{1}{\log \varepsilon_0} \displaystyle\sum_{n=1}^{D-1} \left(\dfrac{D}{n}\right) \log \sin \dfrac{\pi n}{D} & \text{for} \quad D > 0\,, \end{cases}$$

where D again is a fundamental discriminant.

Kronecker's version of the theory of binary quadratic forms has the great advantage of laying the bridge to the theory of quadratic fields: Whenever D is a fundamental discriminant, the classes of binary quadratic forms of discriminant D correspond bijectively to the equivalence classes (in the narrow sense) of ideals in $\mathbb{Q}(\sqrt{D})$. Hence Dirichlet's class number formula may be understood as a formula for the ideal class number of $\mathbb{Q}(\sqrt{D})$, and the gate to the class number formula for arbitrary number fields opens up.

Special cases of Dirichlet's class number formula were already observed by Jacobi in 1832 ([**J.1**], pp. 240–244 and pp. 260–262). Jacobi considered the forms $x^2 + py^2$, where $p \equiv 3 \pmod 4$ is a prime number, and computing both sides of the class number formula, he stated the coincidence for $p = 7, \ldots, 103$ and noted that $p = 3$ is an exceptional case. Only after Gauß' death did it become known from his papers that he had known the class number formula already for some time. Gauß' notes are published in [**G.1**], pp. 269–291 with commentaries by Dedekind (ibid., pp. 292–303); see also Bachmann's report [**Ba.3**], pp. 51–53. In a letter to Dirichlet of November 2, 1838, Gauß deeply regretted that unfortunate circumstances had not allowed him to elaborate on his theory of class numbers of quadratic forms which he possessed as early as 1801 ([**Bi.9**], p. 165).

In another great memoir ([**D.1**], pp. 533–618), Dirichlet extends the theory of quadratic forms and his class number formula to the ring of Gaussian integers $\mathbb{Z}[i]$. He draws attention to the fact that in this case the formula for the class number depends on the division of the lemniscate in the same way as it depends on the division of the circle in the case of rational integral forms with positive determinant (i.e., with negative discriminant; see [**D.1**], pp. 538, 613, 621). Moreover, he promised that the details were to appear in the second part of his memoir, which however never came out.

Comparing the class numbers in the complex and the real domains Dirichlet concluded that

$$H(D) = \xi h(D)h(-D)$$

where D is a rational integral non-square determinant (in Dirichlet's notation of quadratic forms), $H(D)$ is the complex class number, and $h(D), h(-D)$ are the real ones. The constant ξ equals 2 whenever Pell's equation $t^2 - Du^2 = -1$ admits a solution in rational integers, and $\xi = 1$ otherwise. For Dirichlet, "this result ... is one of the most beautiful theorems on complex integers and all the more surprising since in the theory of rational integers there seems to be no connection between forms of opposite determinants" ([**D.1**], p. 508 and p. 618). This result of Dirichlet's has been the starting point of vast extensions (see e.g. [**Ba.2**], [**H**], [**He**], No. 8, [**K.4**], [**MC**], [**Si**], [**Wei**]).

D. Dirichlet's Unit Theorem. An algebraic integer is, by definition, a zero of a monic polynomial with integral coefficients. This concept was introduced by Dirichlet in a letter to Liouville ([**D.1**], pp. 619–623), but his notion of what Hilbert later called the ring of algebraic integers in a number field remained somewhat imperfect, since for an algebraic integer ϑ he considered only the set $\mathbb{Z}[\vartheta]$ as the ring of integers of $\mathbb{Q}(\vartheta)$. Notwithstanding this minor imperfection, he succeeded in determining the structure of the unit group of this ring in his poineering memoir *Zur Theorie der complexen Einheiten* (On the theory of complex units, [**D.1**], pp. 639–644). His somewhat sketchy account was later carried out in detail by his student Bachmann in the latter's *Habilitationsschrift* in Breslau ([**Ba.1**]; see also [**Ba.2**]).

In the more familiar modern notation, the unit theorem describes the structure of the group of units as follows: Let K be an algebraic number field with r_1 real and $2r_2$ complex (non-real) embeddings and ring of integers $\mathfrak{o}_K$. Then the group of units of $\mathfrak{o}_K$ is equal to the direct product of the (finite cyclic) group $E(K)$ of roots of unity contained in K and a free abelian group of rank $r := r_1 + r_2 - 1$. This means: There exist r "fundamental units" $\eta_1, \dots, \eta_r$ and a primitive d-th root of unity ζ ($d = |E(K)|$) such that every unit $\varepsilon \in \mathfrak{o}_K$ is obtained precisely once in the form

$$\varepsilon = \zeta^k \eta_1^{n_1} \cdot \ldots \cdot \eta_r^{n_r}$$

with $0 \leq k \leq d-1, n_1, \ldots, n_r \in \mathbb{Z}$. This result is one of the basic pillars of algebraic number theory.

In Dirichlet's approach the ring $\mathbb{Z}[\vartheta]$ is of finite index in the ring of all algebraic integers (in the modern sense), and the same holds for the corresponding groups of units. Hence the rank r does not depend on the choice of the generating element ϑ of the field $K = \mathbb{Q}(\vartheta)$. (Note that $\mathbb{Z}[\vartheta]$ depends on that choice.)

An important special case of the unit theorem, namely the case $\vartheta = \sqrt{D}$ ($D > 1$ a square-free integer), was known before. In this case the determination of the units comes down to Pell's equation, and one first encounters the phenomenon that all units are obtained by forming all integral powers of a fundamental unit and multiplying these by ± 1. Dirichlet himself extended this result to the case when ϑ satisfies a cubic equation ([**D.1**], pp. 625–632) before he dealt with the general case.

According to C.G.J. Jacobi the unit theorem is "one of the most important, but one of the thorniest of the science of number theory" ([**J.3**], p. 312, footnote, [**N.1**], p. 123, [**Sm**], p. 99). Kummer remarks that Dirichlet found the idea of proof when listening to the Easter Music in the Sistine Chapel during his Italian journey (1843–1845; see [**D.2**], p. 343).

A special feature of Dirichlet's work is his admirable combination of surprisingly simple observations with penetrating thought which led him to deep results. A striking example of such a simple observation is the so-called *Dirichlet box principle* (also called *drawer principle* or *pigeon-hole principle*), which states that whenever more than n objects are distributed in n boxes, then there will be at least one box containing two objects. Dirichlet gave an amazing application of this most obvious principle in a brief paper ([**D.1**], pp. 633–638), in which he proves the following generalization of a well-known theorem on rational approximation of irrational numbers: *Suppose that the real numbers $\alpha_1, \ldots, \alpha_m$ are such that $1, \alpha_1, \ldots, \alpha_m$ are linearly independent over $\mathbb{Q}$. Then there exist infinitely many integral $(m+1)$-tuples $(x_0, x_1, \ldots, x_m)$ such that $(x_1, \ldots, x_m) \neq (0, \ldots, 0)$ and*

$$|x_0 + x_1\alpha_1 + \ldots + x_m\alpha_m| < (\max_{1 \leq j \leq m} |x_j|)^{-m} .$$

Dirichlet's proof: Let n be a natural number, and let $x_1, \ldots, x_m$ independently assume all $2n+1$ integral values $-n, -n+1, \ldots, 0, \ldots, n-1, n$. This gives $(2n+1)^m$ fractional parts $\{x_1\alpha_1 + \ldots + x_m\alpha_m\}$ in the half open unit interval $[0, 1[$. Divide $[0, 1[$ into $(2n)^m$ half-open subintervals of equal length $(2n)^{-m}$. Then two of the aforementioned points belong to the same subinterval. Forming the difference of the corresponding $\mathbb{Z}$-linear combinations, one obtains integers $x_0, x_1, \ldots, x_m$, such that $x_1, \ldots, x_m$ are of absolute value at most $2n$ and not all zero and such that

$$|x_0 + x_1\alpha_1 + \ldots + x_m\alpha_m| < (2n)^{-m} .$$

Since n was arbitrary, the assertion follows. As Dirichlet points out, the approximation theorem quoted above is crucial in the proof of the unit theorem because it implies that r independent units can be found. The easier part of the theorem, namely that the free rank of the group of units is at most r, is considered obvious by Dirichlet.

E. Dirichlet's Principle. We pass over Dirichlet's valuable work on definite integrals and on mathematical physics in silence ([**Bu**]), but cannot neglect mentioning the so-called *Dirichlet Principle*, since it played a very important role in the history of analysis (see [**Mo**]). *Dirichlet's Problem* concerns the following problem: Given a (say, bounded) domain $G \subset \mathbb{R}^3$ and a continuous real-valued function f on the (say, smooth) boundary ∂G of G, find a real-valued continuous function u, defined on the closure $\overline{G}$ of G, such that u is twice continuously differentiable on G and satisfies Laplace's equation

$$\Delta u = 0 \quad \text{on } G$$

and such that $u \,|\, \partial G = f$. Dirichlet's Principle gives a deceptively simple method of how to solve this problem: Find a function $v : \overline{G} \to \mathbb{R}$, continuous on $\overline{G}$ and continuously differentiable on G, such that $v \,|\, \partial G = f$ and such that Dirichlet's integral

$$\int_G (v_x^2 + v_y^2 + v_z^2)\, dx\, dy\, dz$$

assumes its minimum value. Then v solves the problem.

Dirichlet's name was attributed to this principle by Riemann in his epoch-making memoir on Abelian functions (1857), although Riemann was well aware of the fact that the method already had been used by Gauß in 1839. Likewise, W. Thomson (Lord Kelvin of Largs, 1824–1907) made use of this principle in 1847 as was also known to Riemann. Nevertheless he named the principle after Dirichlet, "because Professor Dirichlet informed me that he had been using this method in his lectures (since the beginning of the 1840's (if I'm not mistaken))" ([**EU**], p. 278).

Riemann used the two-dimensional version of Dirichlet's Principle in a most liberal way. He applied it not only to plane domains but also to quite arbitrary domains on Riemann surfaces. He did not restrict to sufficiently smooth functions, but admitted singularities, e.g. logarithmic singularities, in order to prove his existence theorems for functions and differentials on Riemann surfaces. As Riemann already pointed out in his doctoral thesis (1851), this method "opens the way to investigate certain functions of a complex variable independently of an [analytic] expression for them", that is, to give existence proofs for certain functions without giving an analytic expression for them ([**EU**], p. 283).

From today's point of view the naïve use of Dirichlet's principle is open to serious doubt, since it is by no means clear that there exists a function v satisfying the boundary condition for which the infimum value of Dirichlet's integral is actually attained. This led to serious criticism of the method in the second half of the nineteenth century discrediting the principle. It must have been a great relief to many mathematicians when D. Hilbert (1862–1943) around the turn of the 20th century proved a precise version of Dirichlet's Principle which was sufficiently general to allow for the usual function-theoretic applications.

There are only a few brief notes on the calculus of probability, the theory of errors and the method of least squares in Dirichlet's collected works. However, a considerable number of unpublished sources on these subjects have survived which have been evaluated in [**F**].

9. Friendship with Jacobi

Dirichlet and C.G.J. Jacobi got to know each other in 1829, soon after Dirichlet's move to Berlin, during a trip to Halle, and from there jointly with W. Weber to Thuringia. At that time Jacobi held a professorship in Königsberg, but he used to visit his family in Potsdam near Berlin, and he and Dirichlet made good use of these occasions to see each other and exchange views on mathematical matters. During their lives they held each other in highest esteem, although their characters were quite different. Jacobi was extroverted, vivid, witty, sometimes quite blunt; Dirichlet was more introvert, reserved, refined, and charming. In the preface to his tables *Canon arithmeticus* of 1839, Jacobi thanks Dirichlet for his help. He might have extended his thanks to the Dirichlet family. To check the half a million numbers, also Dirichlet's wife and mother, who after the death of her husband in 1837 lived in Dirichlet's house, helped with the time-consuming computations (see [**Ah.2**], p. 57).

When Jacobi fell severely ill with *diabetes mellitus*, Dirichlet travelled to Königsberg for 16 days, assisted his friend, and "developed an eagerness never seen at him before", as Jacobi wrote to his brother Moritz Hermann ([**Ah.2**], p. 99). Dirichlet got a history of illness from Jacobi's physician, showed it to the personal physician of King Friedrich Wilhelm IV, who agreed to the treatment, and recommended a stay in the milder climate of Italy during wintertime for further recovery. The matter was immediately brought to the King's attention by the indefatigable A. von Humboldt, and His Majesty on the spot granted a generous support of 2000 talers towards the travel expenses.

Jacobi was happy to have his doctoral student Borchardt, who just had passed his examination, as a companion, and even happier to learn that Dirichlet with his family also would spend the entire winter in Italy to stengthen the nerves of his wife. Steiner, too, had health problems, and also travelled to Italy. They were accompanied by the Swiss teacher L. Schläfli (1814–1895), who was a genius in languages and helped as an interpreter and in return got mathematical instruction from Dirichlet and Steiner, so that he later became a renowned mathematician. Noteworthy events and encounters during the travel are recorded in the letters in [**Ah.2**] and [**H.1**]. A special highlight was the audience of Dirichlet and Jacobi with Pope Gregory XVI on December 28, 1843 (see [**Koe**], p. 317 f.).

In June 1844, Jacobi returned to Germany and got the "transfer to the Academy of Sciences in Berlin with a salary of 3000 talers and the permission, without obligation, to give lectures at the university" ([**P**], p. 27). Dirichlet had to apply twice for a prolongation of his leave because of serious illness. Jacobi proved to be a real friend and took Dirichlet's place at the Military School and at the university and thus helped him to avoid heavy financial losses. In spring 1845 Dirichlet returned to Berlin. His family could follow him only a few months later under somewhat dramatic circumstances with the help of the Hensel family, since in February 1845 Dirichlet's daughter Flora was born in Florence.

In the following years, the contacts between Dirichlet and Jacobi became even closer; they met each other virtually every day. Dirichlet's mathematical rigour was legendary already among his contemporaries. When in 1846 he received a most prestigious call from the University of Heidelberg, Jacobi furnished A. von Humboldt with arguments by means of which the minister should be prompted to improve upon Dirichlet's conditions in order to keep him in Berlin. Jacobi explained (see [**P**], p. 99): "In science, Dirichlet has two features which constitute his speciality. He alone, not myself, not Cauchy, not Gauß knows what a perfectly rigorous mathematical proof is. When Gauß says he has *proved* something, it is highly probable to me, when Cauchy says it, one may bet as much pro as con, when Dirichlet says it, it is *certain*; *I* prefer not at all to go into such subtleties. *Second*, Dirichlet has created a *new branch* of mathematics, the *application of the infinite series, which Fourier introduced into the theory of heat, to the investigation of the properties of the prime numbers*... Dirichlet has preferred to occupy himself mainly with such topics, which offer the greatest difficulties ..." In spite of several increases, Dirichlet was still not yet paid the regular salary of a full professor in 1846; his annual payment was 800 talers plus his income from the Military School. After the call to Heidelberg the sum was increasesd by 700 talers to 1500 talers per year, and Dirichlet stayed in Berlin — with the teaching load at the Military School unchanged.

10. Friendship with Liouville

Joseph Liouville (1809–1882) was one of the leading French mathematicians of his time. He began his studies at the *École Polytechnique* when Dirichlet was about to leave Paris and so they had no opportunity to become acquainted with each other during their student days. In 1833 Liouville began to submit his papers to Crelle. This brought him into contact with mathematics in Germany and made him aware of the insufficient publication facilities in his native country. Hence, in 1835, he decided to create a new French mathematical journal, the *Journal de Mathématiques Pures et Appliquées*, in short, *Liouville's Journal*. At that time, he was only a 26-year-old *répétiteur* (coach). The journal proved to be a lasting success. Liouville directed it single-handedly for almost 40 years — the journal enjoys a high reputation to this day.

In summer 1839 Dirichlet was on vacation in Paris, and he and Liouville were invited for dinner by Cauchy. It was probably on this occasion that they made each other's acquaintance, which soon developed into a devoted friendship. After his return to Berlin, Dirichlet saw to it that Liouville was elected a corresponding member[6] of the Academy of Sciences in Berlin, and he sent a letter to Liouville suggesting that they should enter into a scientific correspondence ([**Lü**], p. 59 ff.). Liouville willingly agreed; part of the correspondence was published later ([**T**]). Moreover, during the following years, Liouville saw to it that French translations of many of Dirichlet's papers were published in his journal. Contrary to Kronecker's initial plans, not all of these translations were printed in [**D.1**], [**D.2**]; the missing items are listed in [**D.2**], pp. 421–422.

The friendship of the two men was deepened and extended to the families during Dirichlet's visits to Liouville's home in Toul in fall of 1853 and in March 1856, when Dirichlet utilized the opportunity to attend a meeting of the French Academy of

[6] He became an external member in 1876.

Sciences in the capacity of a foreign member to which he had been elected in 1854. On the occasion of the second visit, Mme Liouville bought a dress for Mrs Dirichlet, *"la fameuse robe qui fait toujours l'admiration de la société de Gœttingue"*, as Dirichlet wrote in his letter of thanks ([**T**], *Suite*, p. 52).

Mme de Blignières, a daughter of Liouville, remembered an amusing story about the long discussions between Dirichlet and her father ([**T**], p. 47, footnote): Both of them had a lot of say; how was it possible to limit the speaking time fairly? Liouville could not bear lamps, he lighted his room by wax and tallow candles. To measure the time of the speakers, they returned to an old method that probably can be traced back at least to medieval times: They pinned a certain number of pins into one of the candles at even distances. Between two pins the speaker had the privilege not to be interrupted. When the last pin fell, the two geometers went to bed.

11. Vicissitudes of Life

After the deaths of Abraham Mendelssohn Bartholdy in 1835 and his wife Lea in 1842, the Mendelssohn house was first conducted as before by Fanny Hensel, with Sunday music and close contacts among the families of the siblings, with friends and acquaintances. Then came the catastrophic year 1847: Fanny died completely unexpectedly of a stroke, and her brother Felix, deeply shocked by her premature death, died shortly thereafter also of a stroke. Sebastian Hensel, the under-age son of Fanny and Wilhelm Hensel, was adopted by the Dirichlet family. To him we owe interesting first-hand descriptions of the Mendelssohn and Dirichlet families ([**H.1**], [**H.2**]).

Then came the March Revolution of 1848 with its deep political impact. King Friedrich Wilhelm IV proved to be unable to handle the situation, the army was withdrawn, and a civic guard organized the protection of public institutions. Riemann, at that time a student in Berlin, stood guard in front of the Royal Castle of Berlin. Dirichlet with an old rifle guarded the palace of the Prince of Prussia, a brother to the King, who had fled (in fear of the guillotine); he later succeeded the King, when the latter's mental disease worsened, and ultimately became the German Kaiser Wilhelm I in 1871.

After the revolution the reactionary circles took the revolutionaries and other people with a liberal way of thinking severely to task: Jacobi suffered massive pressure, the conservative press published a list of liberal professors: "The red contingent of the staff is constituted by the names ..." (there follow 17 names, including Dirichlet, Jacobi, Virchow; see [**Ah.2**], p. 219). The Dirichlet family not only had a liberal way of thinking, they also acted accordingly. In 1850 Rebecka Dirichlet helped the revolutionary Carl Schurz, who had come incognito, to free the revolutionary G. Kinkel from jail in Spandau ([**Lac**], pp. 244–245). Schurz and Kinkel escaped to England; Schurz later became a leading politician in the USA.

The general feeling at the Military School changed considerably. Immediately after the revolution the school was closed down for some time, causing a considerable loss in income for Dirichlet. When it was reopened, a reactionary spirit had spread among the officers, and Dirichlet no longer felt at ease there.

A highlight in those strained times was the participation of Dirichlet and Jacobi in the celebration of the fiftieth anniversary jubilee of the doctorate of Gauß in Göttingen in 1849. Jacobi gave an interesting account of this event in a letter to his brother ([**Ah.2**], pp. 227–228); for a general account see [**Du**], pp. 275–279. Gauß was in an elated mood at that festivity and he was about to light his pipe with a pipe-light of the original manuscript of his *Disquisitiones arithmeticae*. Dirichlet was horrified, rescued the paper, and treasured it for the rest of his life. After his death the sheet was found among his papers.

The year 1851 again proved to be a catastrophic one: Jacobi died quite unexpectedly of smallpox the very same day, that little Felix, a son of Felix Mendelssohn Bartholdy, was buried. The terrible shock of these events can be felt from Rebecka's letter to Sebastian Hensel ([**H.2**], pp. 133–134). On July 1, 1952, Dirichlet gave a most moving memorial speech to the Academy of Sciences in Berlin in honour of his great colleague and intimate friend Carl Gustav Jacob Jacobi ([**D.5**]).

12. Dirichlet in Göttingen

When Gauß died on February 23, 1855, the University of Göttingen unanimously wanted to win Dirichlet as his successor. It is said that Dirichlet would have stayed in Berlin, if His Majesty would not want him to leave, if his salary would be raised and if he would be exempted from his teaching duties at the Military School ([**Bi.7**], p. 121, footnote 3). Moreover it is said that Dirichlet had declared his willingness to accept the call to Göttingen and that he did not want to revise his decision thereafter. Göttingen acted faster and more efficiently than the slow bureaucracy in Berlin. The course of events is recorded with some regret by Rebecka Dirichlet in a letter of April 4, 1855, to Sebastian Hensel ([**H.2**], p. 187): "Historically recorded, ... the little Weber came from Göttingen as an extraordinarily authorized person to conclude the matter. Paul [Mendelssohn Barthody, Rebecka's brother] and [G.] Magnus [1802–1870, physicist in Berlin] strongly advised that Dirichlet should make use of the call in the manner of professors, since nobody dared to approach the minister before the call was available in black and white; however, Dirichlet did not want this, and I could not persuade him with good conscience to do so."

In a very short time, Rebecka rented a flat in Göttingen, Gotmarstraße 1, part of a large house which still exists, and the Dirichlet family moved with their two younger children, Ernst and Flora, to Göttingen. Rebecka could write to Sebastian Hensel: "Dirichlet is contentissimo" ([**H.2**], p. 189). One year later, the Dirichlet family bought the house in Mühlenstraße 1, which still exists and bears a memorial tablet. The house and the garden (again with a pavillon) are described in the diaries of the Secret Legation Councillor K.A. Varnhagen von Ense (1785–1858), a friend of the Dirichlets', who visited them in Göttingen. Rebecka tried to renew the old glory of the Mendelssohn house with big parties of up to 60–70 persons, plenty of music with the outstanding violinist Joseph Joachim and the renowned pianist Clara Schumann performing — and with Dedekind playing waltzes on the piano for dancing.

Dirichlet rapidly felt very much at home in Göttingen and got into fruitful contact with the younger generation, notably with R. Dedekind and B. Riemann (at that time assistant to W. Weber), who both had achieved their doctor's degree and *Habilitation* under Gauß. They both were deeply grateful to Dirichlet for the stimulance and assistance he gave them. This can be deduced from several of Dedekind's letters to members of his family (e.g. [**Sch**], p. 35): "Most useful for me is my contact with Dirichlet almost every day from whom I really start learning properly; he is always constantly kind to me, tells me frankly which gaps I have to fill in, and immediately gives me instructions and the means to do so." And in another letter (ibid., p. 37) we read the almost prophetic words: "Moreover, I have much contact with my excellent colleague Riemann, who is beyond doubt after or even with Dirichlet the most profound of the living mathematicians and will soon be recognized as such, when his modesty allows him to publish certain things, which, however, temporarily will be understandable only to few." Comparing, e.g. Dedekind's doctoral thesis with his later pioneering deep work one may well appreciate his remark, that Dirichlet "made a new human being" of him ([**Lo**], p. 83). Dedekind attended all of Dirichlet's lectures in Göttingen, although he already was a *Privatdozent*, who at the same time gave the presumably first lectures on Galois theory in the history of mathematics. Clearly, Dedekind was the ideal editor for Dirichlet's lectures on number theory ([**D.6**]).

Riemann already had studied with Dirichlet in Berlin 1847–1849, before he returned to Göttingen to finish his thesis, a crucial part of which was based on Dirichlet's Principle. Already in 1852 Dirichlet had spent some time in Göttingen, and Riemann was happy to have an occasion to look through his thesis with him and to have an extended discussion with him on his *Habilitationsschrift* on trigonometric series in the course of which Riemann got a lot of most valuable hints. When Dirichlet was called to Göttingen, he could provide the small sum of 200 talers payment per year for Riemann which was increased to 300 talers in 1857, when Riemann was advanced to the rank of associate professor.

There can be no doubt that the first years in Göttingen were a happy time for Dirichlet. He was a highly esteemed professor, his teaching load was much less than in Berlin, leaving him more time for research, and he could gather around him a devoted circle of excellent students. Unfortunately, the results of his research of his later years have been almost completely lost. Dirichlet had a fantastic power of concentration and an excellent memory, which allowed him to work at any time and any place without pen and paper. Only when a work was fully carried out in his mind, did he most carefully write it up for publication. Unfortunately, fate did not allow him to write up the last fruits of his thought, about which we have only little knowledge (see [**D.2**], p. 343 f. and p. 420).

When the lectures of the summer semester of the year 1858 had come to an end, Dirichlet made a journey to Montreux (Switzerland) in order to prepare a memorial speech on Gauß, to be held at the Göttingen Society of Sciences, and to write up a work on hydrodynamics. (At Dirichlet's request, the latter work was prepared for publication by Dedekind later; see [**D.2**], pp. 263–301.) At Montreux he suffered a heart attack and returned to Göttingen mortally ill. Thanks to good care he seemed to recover. Then, on December 1, 1858, Rebecka died all of a sudden and completely unexpectedly of a stroke. Everybody suspected that Dirichlet would not for long survive this turn of fate. Sebastian Hensel visited his uncle for the last time on Christmas 1858 and wrote down his feelings later ([**H.2**], p. 311 f.): "Dirichlet's condition was hopeless, he knew precisely how things were going for him, but he faced death calmly, which was edifying to observe. And now the poor Grandmother! Her misery ... to lose also her last surviving child, ... was terrible to observe. It was obvious that Flora, the only child still in the house, could not stay there. I took her to Prussia ..." Dirichlet died on May 5, 1859, one day earlier than his faithful friend Alexander von Humboldt, who died on May 6, 1859, in his 90th year of life. The tomb of Rebecka and Gustav Lejeune Dirichlet in Göttingen still exists and will soon be in good condition again, when the 2006 restorative work is finished. Dirichlet's mother survived her son for 10 more years and died only in her 100th year of age. Wilhelm Weber took over the guardianship of Dirichlet's under-age children ([**Web**], p. 98).

The Academy of Sciences in Berlin honoured Dirichlet by a formal memorial speech delivered by Kummer on July 5, 1860 ([**Ku**]). Moreover, the Academy ordered the edition of Dirichlet's collected works. The first volume was edited by L. Kronecker and appeared in 1889 ([**D.1**]). After Kronecker's death, the editing of the second volume was completed by L. Fuchs and it appeared in 1897 ([**D.2**]).

Conclusion

Henry John Stephen Smith (1826–1883), Dublin-born Savilian Professor of Geometry in the University of Oxford, was known among his contemporaries as the most distinguished scholar of his day at Oxford. In 1858 Smith started to write a report on the theory of numbers beginning with the investigations of P. de Fermat and ending with the then (1865) latest results of Kummer, Kronecker, and Hurwitz. The six parts of Smith's report appeared over the period of 1859 to 1865 and are very instructive to read today ([**Sm**]). When he prepared the first part of his report, Smith got the sad news of Dirichlet's death, and he could not help adding the following footnote to his text ([**Sm**], p. 72) appreciating Dirichlet's great service to number theory: "The death of this eminent geometer in the present year (May 5, 1859) is an irreparable loss to the science of arithmetic. His original investigations have probably contributed more to its advancement than those of any other writer since the time of Gauss, if, at least, we estimate results rather by their importance than by their number. He has also applied himself (in several of his memoirs) to give an elementary character to arithmetical theories which, as they appear in the work of Gauss, are tedious and obscure; and he has done much to *popularize* the theory of numbers among mathematicians — a service which is impossible to appreciate too highly."

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