# 8 Super Ultimate Relay

#### 8.1 The Rules

- $A_n$  is the answer to problem *n*. Answers are not necessarily integers.
- The final answer is a name.
- Map is on the back. Good luck!

#### 8.2 The Problems

- 1. What is the only odd two-digit number with exactly five factors?
- 2. What is the period of the repeating decimal  $\frac{1}{A_1}$ ?
- 3. Let  $P(x) = (x+1)^{A_2}$ . When P(x) is expanded, a polynomial  $a_0 + a_1 x + a_2 x^2 + \dots + a_{A_2} x^{A_2}$  is obtained. What is  $\sum_{i=0}^{A_2} a_i$ , i.e. the sum of the coefficients of P(x)?
- 4. Let  $x = \sqrt{A_3}$ . What is (x+i)(x-i)?
- 5. k is a real number such that  $x = A_7$  is one real solution to  $\sqrt{x} + \sqrt{20 x} = k$ . What is the other real solution?
- 6. Let  $\Omega$  be a circle centered at O with radius 20.19. Let P and Q be points on the circumference of  $\Omega$ , and let  $\omega$  be a circle with diameter  $\overline{PO}$ . Suppose  $\overline{PQ}$  intersects  $\omega$  again at R. If  $RQ = A_5$ , what is PQ?
- 7. What is  $\sqrt{A_6^2 2A_7 1}$ ?
- 8. There are  $A_5 + A_7$  runners on a 2018-meter circular track with exactly one lane.  $A_5$  runners start running clockwise, while the other  $A_7$  runners start running counterclockwise. All runners run at the same pace, and when two runners meet they collide, with both runners turning around and running in the other direction. The instant all runners have run more than 1009 meters, there have been how many collisions?
- 9. wxv9Dwqd5nizAzdFTsmpFSxaLrL4dvuyE5VIVbk8lx9Pv8OkshedK8As4pqbkBNTv0gwVG61VPH9
- 10. Let p be the greatest prime factor of  $A_9$ . Compute the integer  $\sqrt{4 \operatorname{gcd}(375, A_9) p}$ .
- 11. The number  $\sqrt{A_9}$  has three digits. The first two digits sum to the third digit. What is the greatest exponent in the prime factorization of  $A_9$ ?
- 12. What is  $\log_2 \left( 2^{A_{10}} \log_{A_{11}}(A_{10}) 2^{A_{11}} \log_{A_{10}}(A_{11}) \right)$ ?
- 13. In  $\triangle ABC$ , *M* is the midpoint of *AC*. A point *P* is selected on segment *BM* such that BP = 2PM. Ray *AP* intersects *BC* at *N*. Given that  $AN = A_9$ , what is *PN*?

- 14. A particle starts at the origin, and moves around along the lattice coordinates in one-unit steps. An X-move is a movement left or right one unit, and a Y-move is a movement up or down one unit. Let M and N be integers. Then if the particle makes M X-moves and N Y-moves, there are  $A_{13}$  possible coordinates the particle could end up at. If  $M, N \ge 0$ , how many possible values of M are there?
- 15. Let  $k = \frac{A_{12}}{A_{14}}$ . Let f(x) be a real-valued function satisfying  $f(x) = A_{14}x$  for all x in the interval  $-k \le x \le k$ , and satisfying f(a) = f(b) for all pairs (a, b) satisfying either a + b = -2k or a + b = 2k. The region R consists of all points (x, y) satisfying  $0 \le x \le A_{12}$  and  $f(x) \le y \le A_{12}$ . What is the area of R?

## 8.3 Map

To solve for a certain answer, you must first get all answers pointing to it!



### 8.4 Finale

It's time. Compute  $A_4 \cdot 10^6 + A_8 \cdot 10^3 + A_{15}$ .



Who is my best friend?