Mock AMC 10

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Wednesday April 1st, 2020

1. Please take 30 minutes to complete the following 10 questions.
2. Scoring is as follows:
   (a) 6 points for each correct answer
   (b) 1.5 points for each blank answer
   (c) 0 points for each wrong answer

   Thus, the AMC 10 is scored out of 60.
3. Keep in mind that this is a *very* fair and accurate test that will definitely be a good measure of your mathematical ability.
4. Good luck!

1. Let O be the circumcenter of $\triangle ABC$. If the area of $\triangle ABC$ is 15, the area of $\triangle ABO$ is 3, and the area of $\triangle ACO$ is 8, what is the area of $\triangle BCO$?

   (A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

2. Me and nine friends are throwing the first of many parties!
   But before the fun starts, there are formalities.
   There are some old that I'd like to re-greet,
   And some new that I'd like to meet.

   Therefore, I say,
   It'd be good to shake hands in some way,
   So our friendships shalt not worsen.
   But this is a math problem, you see.
   So I (as well as everyone else) will only shake hands with a person,
   If that person is shorter than me.

   If we shake all the hands we can, how many hands were shaken in total?
   (Your answer better be a reasonable proposal!)

   (A) 0 **(B)** 45 **(C)** 55 **(D)** 90 **(E)** 110

3. Anne and Sue keep getting their bikes stolen, so they are making chains by linking metal links together. Assume that Anne and Sue work at constant rates. If Anne can make a chain of 5 links in one hour, and Sue can make a chain of 10 links in one hour, how long will it take them to make a chain of 300 links if they work together, in *hours*?

   (A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

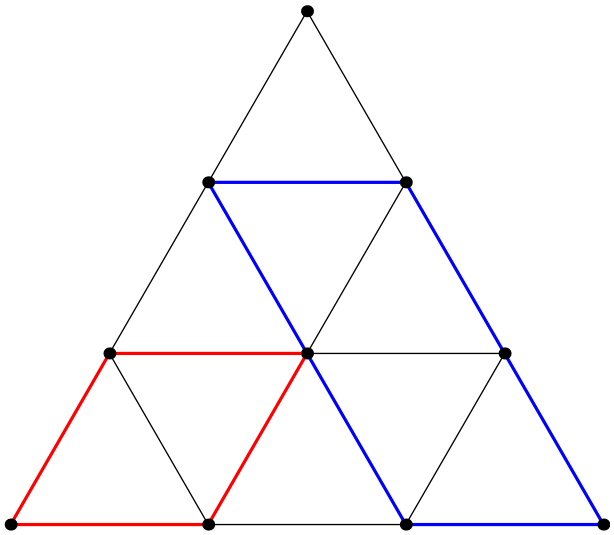
4. What is the last digit of $11^{10} - 10^{11}$, written as a base 10 numeral?

   (A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

5. A bunny is 80 feet away from its rabbit hole! The bunny can only hop either 5 feet or 9 feet at a time. What is the least number of hops it needs to get back to the rabbit hole? The rabbit hole is a hole in the ground, so hopping over it won't do.

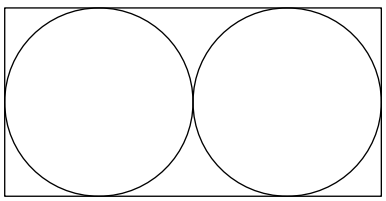
   (A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

6. Ten points are arranged in an equilateral triangular lattice. How many parallelograms of positive area can be formed by connecting four of these ten points? The lattice and two example parallelograms are shown below.



(A) 12 **(B)** 15 **(C)** 18 **(D)** 21 **(E)** 24

7. A rectangle with side lengths 2 and 4 is the rectangle of least area that can completely contain two circles of radius 1 such that the circles are either tangent or non-intersecting, as shown below.



But finding such a rectangle may be more difficult for other radii. The area of the rectangle of least area that can completely contain two circles of radii 1 and 2 such that the circles are either tangent or non-intersecting can be written in the form $a+b\sqrt{c}$, where a, b, and c are positive integers such that c has no perfect square factors other than 1. What is $a+b+c$?

(A) 17 **(B)** 19 **(C)** 22 **(D)** 24 **(E)** 25

8. What is the value of the sum

$$\sum_{n=1}^{\infty} \sqrt{1+\frac{1}{n}} - \sqrt{1+\frac{1}{n+1}}?$$

(A) $\sqrt{2}-1$ **(B)** $\sqrt{2}$ **(C)** $\sqrt{2}+1$ **(D)** $\sqrt{3}$ **(E)** $\sqrt{3}+1$

9. For an integer a, let $f(a)$ be the sum of all complex values of x that satisfy the equation $\frac{1}{ax-2^a x^2} = 16$. Suppose that the sum

$$\sum_{a=0}^{\infty} f(a)$$

can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

(A) 3 **(B)** 4 **(C)** 11 **(D)** 47 **(E)** 191

10. In triangle $\triangle ABC$, it is given that $\sin\angle B = \frac{4}{5}$, $\sin\angle C = \frac{3}{5}$, and $BC = 5$. A point P is chosen on the incircle and a point Q is chosen on the circumcircle. What is the maximum possible value of PQ?

(A) $\frac{3+\sqrt{5}}{2}$ **(B)** $\frac{7+\sqrt{5}}{2}$ **(C)** $\frac{145+25\sqrt{17}}{14}$ **(D)** $\frac{35+98\sqrt{19}}{26}$ **(E)** $\frac{29+5\sqrt{17}}{2}$