Math-UA.233.001: Theory of Probability
Midterm cheatsheet

The midterm will be a closed-book exam, so there are some facts from the course that you'll be expected to remember. It will not be assumed that you remember everything from all the classes. Try to make sure you know the following. [I’ve put the relevant section numbers from Ross in square brackets.]

Pre-requisites about sets and counting:

- The Basic Principle of Counting. [1.2]
- Permutations. Factorials: number of permutations of $n$ objects. [1.3]
- Ordered choices: number of ways to choose a list of $r$ objects from a set of $n$ objects. [1.3]
- How to count permutations of a set of $n = n_1 + \cdots + n_r$ objects in which the first $n_1$ are indistinguishable from one another (so the order among those first $n_1$ doesn’t matter), the next $n_2$ are indistinguishable from one another, \ldots, and the last $n_r$ are indistinguishable from one another. [1.3]
- Combinations. Binomial coefficients: number of ways to choose $r$ from $n$ objects when order doesn’t matter. [1.4]
- Multinomial coefficients: number of ways to separate $n = n_1 + \cdots + n_r$ objects into a group of size $n_1$, a group of size $n_2$, \ldots, and a group of size $n_r$. [1.5]

Definitions:

- ‘Experiment’, ‘sample space’, ‘outcome’ and ‘event’. (Outcomes are not the same as events!) [2.2]
- The ‘null event’. [2.2]
- Events that are ‘mutually exclusive’. [2.2]
• The axioms of probability. [2.3]
• The ‘uniform distribution’ on a finite sample space, also known as the distribution of ‘equally likely outcomes’. [2.5]
• The distribution of a $p$-biased coin. [e.g. 4.6]
• ‘Conditional probability of one event given another’ (and what this means in terms of updating probability values in the light of new information). [3.2]
• ‘Partitions’ of the sample space.
• Two or more events being ‘independent’. (I will be very upset with anyone who confuses the notions ‘mutually exclusive’ and ‘independent’!) [3.4]
• ‘Random variables’ (‘RV’s). [4.1]
• The ‘cumulative distribution function’ (‘CDF’) of a RV. [4.1]
• A RV being ‘discrete’. [4.2]
• The ‘probability mass function’ (‘PMF’) of a discrete RV. [4.2]
• The ‘expectation’ and ‘variance’ of a discrete RV. [4.3 and 4.5]
• The ‘indicator variable’ of an event. [4.3]
• A ‘Bernoulli process of $n$ trials with bias $p$’. [e.g. 4.6]
• Important distributions of discrete random variables:
  – Bernoulli($p$) [4.6];
  – Binom($n, p$) [4.6];
  – Poi($\lambda$) [4.7].

[If you need another distribution, such as a hypergeometric distribution, in an exam, it will be given to you in some form.]

**Ideas, facts, propositions:**

• Basic operations with sets/events: ‘∩’ (‘intersection’), ‘∪’ (‘union’) and ‘⊆’ (‘complement’). The relations between them (distributivity, etc.). [2.2]

• De Morgan’s Laws. [2.2]
  – $P(E^c) = 1 - P(E)$;
– if $E \subset F$ then $P(E) \leq P(F)$;
– for any events $E$ and $F$ one has $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

(more complicated relatives of these will be given to you if you need them in an exam). [2.4]

• Conditional probability is consistent with equally likely outcomes. [3.2]

• Re-arrangement of the definition of conditional probability to $P(E \cap F) = P(E \mid F)P(F)$; the Multiplication Rule. [3.2]

• The Law of Total Probability. [3.2]

• Bayes’ Formula [3.3].

• In a Bernoulli trials process, the results of all the trials are independent.

• If an event is defined in terms of the values taken by a discrete RV, then you can compute its probability in terms of the PMF of that RV.

• Computing $E(g(X))$ in terms of the PMF of $X$, where $X$ is a RV and $g$ is a function from real values to real values. [4.4]

• Linearity of Expectation. [4.3 and 4.9]

• $\text{Var}(X) = E(X^2) - (EX)^2$. [4.5]

• Recognizing when it’s natural to assume that a RV has a Poisson($\lambda$) distribution for some $\lambda$, especially because of the Poisson approximation to the binomial. How we find $\lambda$ in practice (usually you are told the expectation of the RV). [4.7]

(If you need another theorem from the course, then it will be given to you in the question.)

The last word:

• If you’re not sure what to do, just write down the relevant definitions. This will often clarify the question, and may be worth marks by itself.