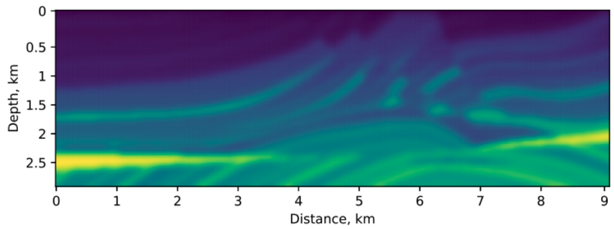
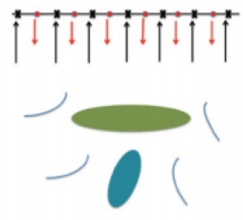
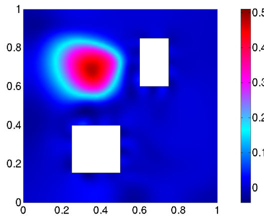
Fall 2021: Advanced Topics in Numerical Analysis: Inverse Problems (+PDE-constrained optimization)

Cross-listed as MATH-GA 2011.001 and CSCI-GA 2945.001







Lectures: Wednesday 5:10–7:00pm, WWH 517 (first class Sep 8)

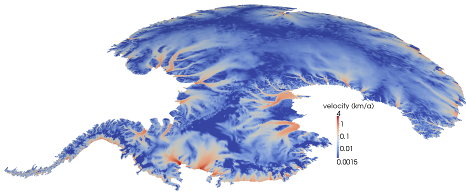
Instructor: Georg Stadler, WWH #929
E-mail: stadler@cims.nyu.edu
Zoom: https://nyu.zoom.us/my/georgstadler
Office hours: TBA, most likely over Zoom

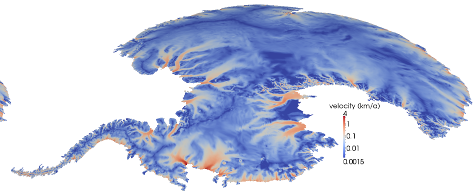
Description: This course provides an introduction to inverse problems that are governed by partial differential equations (PDEs), and to their numerical solution. The focus of the course is on ill-posedness, regularization, computation of gradients using adjoints and large-scale solution algorithms for inverse problems. We will also explore the relation between inverse problems and neural network training. Besides theoretic aspects, we will use numerical implementations for model problems using an inverse problems Python library, which builds on a high-level finite element toolkit, FEniCS. This will allow us to study the influence of data noise, regularization, the observation operator, and the nature of the underlying model on the identifiability of the model parameters, and it will facilitate experimentation with different solution algorithms. Depending on the interest of the participants (and on available time), the course will provide an introduction to the Bayesian framework for inverse problems additionally to the deterministic approach, and will discuss connections between the two. Examples will be drawn from different areas of science and engineering, including image processing and continuum mechanics.

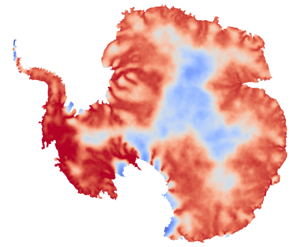
Prerequisites: Graduate linear algebra, familiarity with PDEs and numerical methods. Some of the other required background on nonlinear optimization, variational methods, neural networks and basic probability will be covered as needed. Since we use a library for approximating PDEs, knowledge on the numerical approximation of PDEs is not critical. You should be familiar with either Matlab or Python. Contact me if in doubt.

Required work: About 5 obligatory homework assignments involving a mix of theory and computational experiments/implementation.

Intended topics: Introduction to inverse problems with PDEs; ill-posedness and regularization; regularization techniques; neural network training as inverse problem; variational methods; weak forms; PDE-constrained optimization problems; computing derivatives via adjoints; descent methods from nonlinear optimization, Newton-conjugate gradient method; Bayesian approach to inverse problems, and the relation to uncertainty quantification (time permitting).







Useful References

No textbook required, but several good references for variational inverse problems (which go far beyond the material covered in class) include:

Theory and computational methods for inverse problems:

- Heinz Engl, Michael Hanke, and Andreas Neubauer, *Regularization of Inverse Problems*, Dordrecht, 2nd edition, 1996. (classic book on regularization of inverse problems)
- Curtis R. Vogel, *Computational Methods for Inverse Problems*, SIAM, 2002. (focus on numerical solution methods, applications mainly in image restoration)

Numerical optimization background:

- Jorge Nocedal and Stephen J. Wright, *Numerical Optimization*, Springer-Verlag, 1999. (comprehensive guide, free access to PDF from NYU campus)
- C. Tim Kelley, *Iterative Methods of Optimization*, SIAM, 1999. (unconstrained, lots of practical advice, PDF is available online)
- Phillip E. Gill, Walter Murray and Margaret H. Wright, *Practial Optimization*, Emerald, 1982.

Optimization of systems governed by PDEs:

- Max D. Gunzburger, *Perspectives in Flow Control and Optimization*, SIAM, 2003. (more general than implied by title)
- Fredi Tröltzsch, *Optimal Control of Partial Differential Equations: Theory, Methods and Applications*, Graduate Studies in Mathematics Vol. 112, AMS, 2010.
- M. Hinze, R. Pinnau, M. Ulbich, and S. Ulbrich, *Optimization with PDE constraints*, Springer, 2009. (free access to PDF from NYU campus)
- Alfio Borzi and Volker Schulz, *Computational Optimization of Systems Governed by Partial Differential Equations*, SIAM, 2012.

Probabilistic approach to inverse problems:

- Albert Tarantola, *Inverse Problem Theory and Methods for Model Parameter Estimation*, SIAM, 2005. (statistical perspective; freely available on the web)
- Jari Kaipio and Erkki Somersalo, *Statistical and Computational Inverse Problems*, Springer, 2005. (free access to PDF from NYU campus)
- Jonathan Bardsley, *Computational Uncertainty Quantification for Inverse Problems*, SIAM 2018. (focus on numerical methods for UQ, mostly applications in image processing)