



Methods of Applied Mathematics

MATH-GA 2701
Tuesdays 1:25 - 3:15
CIMS 517
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Description: This is a first-year course for any incoming PhD and Master students interested in pursuing research in applied mathematics. It provides a concise introduction to advanced mathematical methods, especially in the asymptotic analysis of differential equations. Topics include scaling, perturbation methods, integral approximations, matched asymptotics, boundary layers, WKB, multiple scale methods, geometric wave theory and calculus of variations. Assessment will be based on weekly homework assignments and a final exam. Students should have a working knowledge of differential equations, linear algebra and some complex variables.

Reference texts (* recommended; † available online via NYU Bobcat)

- Barenblatt [B] (1996) *Scaling, self-similarity, and intermediate asymptotics*
Bender & Orszag* [BO] (1999) *Advanced mathematical methods for scientists ...*
Bühler [Bu] (2006) *Mechanics*
Gelfand & Fomin [GF] (2000) *Calculus of variations*
Hinch* [H] (1991) *Perturbation methods*
Holmes [Ho] (1995) *Introduction to perturbation methods*
Kevorkian & Cole [KC] (1996) *Multiple scale and singular perturbation methods*
Majda [M] (2003) *Introduction to waves and PDEs for the atmosphere and ocean*
Mathews & Walker [MW] (1964) *Mathematical methods of physics*
Riley, Hobson & Bence*† [RBH] (2006) *Mathematical methods for physics ...*
Strogatz [S] (1994) *Nonlinear dynamics and chaos*
Whitham [W] (1974) *Linear and nonlinear waves*

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Rough schedule of topics

- Scaling, dimensional analysis, similarity solutions (B 1)
- Regular and singular perturbation theory (BO 3, 4; H 1)
- Integral approximations: Integration by parts, Laplace's method, methods of stationary phase and steepest descent (BO 6; H 3)
- Matched asymptotic expansions and boundary layer theory (BO 9; H 5)
- WKB methods and strained coordinates (BO 10; H 6)
- Method of multiple scales (BO 11; H 7)
- Geometric wave theory, group velocity, energy methods (W; Bu)
- Homogenization theory (H 7; Ho 5)
- Calculus of variations and conservation laws (RBH 22; GF; Bu)
- Multiple scale methods for PDEs (H 7; KC; M)

Notes

1. We will review, as necessary, the fundamentals of ordinary differential equations and complex analysis. Key topics in the former are phase plane analysis, integral transforms, series solutions and special functions (BO 1; MW 1; RBH 13, 14, 16, 17; S), and for the latter, analyticity, branch cuts, Cauchy and residue theorems, and conformal mapping (RBH 20).

2. The information in parentheses indicates a text (see keys following list on previous page) and a chapter. These are broad, overlapping references intended to give you some general idea where to find material relevant to the given topic.

3. This list is a rough outline that will be abused and revised as we move through the course.