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## The Cayley graph of $H_{\mathbb{Z}}^{3}$



$$
\begin{aligned}
z & =x y x^{-1} y^{-1} \\
z^{4} & =x^{2} y^{2} x^{-2} y^{-2} \\
z^{n^{2}} & =x^{n} y^{n} x^{-n} y^{-n}
\end{aligned}
$$

From Cayley graph to sub-riemannian metric


Horizontal planes spanned by red and blue edges.

## Horizontal curves in H



A geodesic in $H$

