Problem set 11
Computational Complexity.

1. Solve question 2 in exam cc04c-a, regarding levin-reductions.
2. If PH has a complete problem, then PH collapses to some Σ_k.
3. Solve question 3a from exam cc04b-b.
(hint: thinking about PH may help).
4. In the following question all calculations are done modulo 2.
   (a) Suppose v_1, v_2 are two different linear Boolean vectors of length n. Show that for a random Boolean vector x of length n,
   $$\Pr_x[v_1 \cdot x = v_2 \cdot x] = \frac{1}{2}$$
   (b) We want an algorithm that given three Boolean matrices A, B, C of size $n \times n$ checks whether $AB = C$. This can be easily done in time $O(n^3)$.
   Suggest an $O(n^2)$ time algorithm that answers 'yes' with probability 1 if $AB = C$, and answer 'no' with probability at least $\frac{1}{2}$ if $AB \neq C$.
   (Hint: multiplying a matrix of size $n \times n$ with a vector of length n takes only $O(n^2)$ time).
5. Alice holds $x_1, .., x_n$ where each of them is from $\{1...m\}$. Bob holds $y_1, ..., y_n$ where each of them is from $\{1...m\}$.

   They want to find the k'th element in the sorted list of $x_1, ..., x_n, y_1, ..., y_n$. You may assume no value appears twice.

   (a) Show a deterministic protocol using $O(\log(n) \cdot \log(m))$ communication bits.
   (b) Show a deterministic protocol using only $O(\log(n) + \log(m))$ communication bits.
   Hint: divide the sorted list of each player into subsections of size n/100. On each round use constant number of communication bits to either reduce the number of elements or reduce the range of relevant values (starting with all values from 1 to m).