Problem set 3

Computational Complexity.

1. Show that NL is closed under union and under intersection. That is,
   (a) If $A \in NL$ and $B \in NL$ then $A \cup B \in NL$.
   (b) If $A \in NL$ and $B \in NL$ then $A \cap B \in NL$.
2. Two vertices x, y of a directed graph G are in the same Strongly Connected Component *(SCC)* if there is a path from x to y and from y to x. Prove that the following problems are in NL:
   (a) Input: A directed graph G and to vertices in it u and v.
   Question: Are u and v in the same SCC?
   (b) Input: A directed graph G.
   Question: Does G contains at least 2006 SCCs?
   (c) Input: A directed graph G.
   Question: Does G contains exactly 2006 SCCs?
3. Show that deciding whether a graph G is bipartite is in NL (a graph is bipartite iff there exists a partition of its vertices into two sets so that every edge of the graph has one vertex in each part).
4. Prove that $NP \subseteq NP^*$ (recall that in an NP^* machine all the guesses are made at the beginning).
5. Prove that if $CVAL \in L$ then $P = L$.
6. We want to prove the following claim. There exist Boolean functions on n bits that require circuits of size $\Omega(\frac{2^n}{n})$.
   - Explain how many Boolean functions on n bits are there.
   - Give an upper bound to the number of circuits, containing $c \cdot \frac{2^n}{n}$ gates, where each gate is one of 3 types (AND/OR/NOT) and has at most two other gates connected to its inputs.
   - Conclude the claim, by showing that for small enough constant c, there are much more functions than circuits.