Problem set 9
Computational Complexity.

1. Consider the Steiner tree problem:
Input: A number q, a complete graph $G = (V, E)$, a non-negative weight function $W : E \rightarrow R^+$ and a subset $R \subseteq V$ of required vertices.
Question: Is there a Steiner tree of weight at most q? i.e., is there a subtree T of G that includes all the vertices in R (and may include other vertices as well) so that the sum of the weights of the edges in the subtree T is at most q?

   - Show that this problem is NP-Hard.
   Hint: consider the Vertex-Cover problem. Start with creating a bipartite graph H with $V(G)$ on one side and $E(G)$ on the other side, connecting $v \in V$ to $e \in E$ if v is a vertex of e. Give these edges one weight. Now make H complete, and decide what weights should be given to each of the three types of new edges.
   - Show that your reduction works even when all weights are 1 and 2. Conclude that the Steiner tree problem remains NP-Hard even if triangle inequality holds.
   - Use the hardness of approximation proof we saw for bounded degree Vertex-Cover. Conclude that there exists a constant $c > 1$ such that approximating Steiner Tree (with triangle inequality) to within c is NP-Hard.

2. Let $2-O-3SAT$ be the problem of $3SAT$ where each variable appears exactly twice. Prove that $2-O-3SAT \in P$.
(Hint: consider the bipartite graph where one side is corresponds to the clauses, the other side corresponds to variables, and there is an edge between a clause and a variable iff the variable appears in the clause. Then consider Hall's Theorem).

3. Solve question 4 from exam cc04c-a regarding Min-Un-Cut.

4. Prove the following claims:
   (a) $P^{PSPACE} = PSPACE$
   (b) $NP^{PSPACE} = NPSPACE$
   (c) $PSPACE = NPSPACE$
   (d) $P^{EXP} = EXP$
   (e) $NP^{EXP} = EXP$