Problem set 6
Computational Complexity.

1. Recall that $G = (V, E)$ is c-expander if for every $A \subseteq V$, $|E(A, V \setminus A)| \geq c \cdot \min(|A|, |V \setminus A|)$. We call c the expansion factor of G.

   Find the expansion factor c of the following graphs. Prove both the upper and lower bound on the expansion. Note that c may depend on n. Do not confuse n with the number of vertices.

   (a) K_n. That is, a clique of size n.

   (b) $Z_n \times Z_n$ lattice. That is, $V = (i, j) \in \{1, ..., n\} \times \{1, ..., n\}$ and (i_1, j_1) is a neighbour of (i_2, j_2) if
   $i_1 - i_2 = \pm 1 (mod\ n)$ and $j_1 = j_2$ or
   $j_1 - j_2 = \pm 1 (mod\ n)$ and $i_1 = i_2$.

   (c) The cube Z_2^n. That is $V = \{0, 1\}^n$ and $x = (x_1, ..., x_n)$ is a neighbour of $y = (y_1, ..., y_n)$ if for all but one index we have $x_i = y_i$.

2. Prove that if G is a c-expander with n vertices, for some constant value c then it has a small diameter, i.e, for every two vertices u, v in G the minimal distance between u and v is $O(\log n)$.

3. Show that there exists a constant factor $c < 1$ such that $3NAE$ cannot be approximated to within c unless $P = NP$.

4. Let $EiSAT$ be the variant of SAT where each clause is of size exactly i, with no repetition of variables inside a clause.

   (a) Assuming that the gap shown in class for $3SAT$ holds for $E3SAT$, show the best constant factor $c > 0$ such that $E4SAT$ cannot be approximated to within c unless $P = NP$.

   (b) Prove that this factor is optimal.

   (c) Is it optimal for $4SAT$ too?

5. Solve question 4 from exam cc04b-a, regarding LinEq problem (available on the web-page).