Problem set 5

Computational Complexity.

1. Let $UpToOneSat$ be the following language:
   $UpToOneSat = \{\Phi \mid \Phi \in CNF$, There exists at most one satisfying assignment to $\Phi\}$

   (a) Prove:
   If $NP = coNP$ then $UpToOneSat \in NP$.

   (b) Prove:
   If $UpToOneSat \in NP$ then $NP = coNP$.

2. Assuming $P \neq NP$, does the following problem have a polynomial time algorithm?
   The clique problem, where the degree of every vertex in the graph is exactly 2005.

3. Consider the greedy algorithm for Vertex-Cover problem:
   Choose a vertex of maximal degree. Add it to the cover, and remove it and all its edges from the graph. Repeat until no edges remain in the graph.
   Prove that this algorithm is *not* a 2-approximation.

4. We say that a (not necessarily Boolean) function f is computable in NL if there exists a non-deterministic TM M that has 3 tapes: input (read only), output (write only) and working (read/write). M gets on its input tape x and uses on the working tape a space bounded by $O(\lg |x|)$. On the output tape M should either write an $f(x)$ and move to the state 'accept' or any string and move to the state 'reject'. For every input x there should be at least one run for which M moves to 'accept'.

   Prove that if f and g are computable in NL the so is $f(g)$.

5. Show that MST is computable in NL. That is - the input is a graph G and a weight function $W : E(G) \rightarrow N$ and the output is a minimum spanning tree for G.
   (Hint: recall Kruskal's algorithm).

6. Solve question 3 in exam cc03b-b regarding approximating minimum vertex deletion for tournament graphs (available at the web-page).