Homework 4
Due 2012/10/29

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Solve at least two out of the three questions (or all three for extra credit).

1. **A hardness reduction that fails:** Consider the following attempt to show that for any $\eta > 0$, $(\frac{1}{2}+\eta, 1-\eta)$-MAX3LIN is unique-games-hard. Given a unique CSP $G = (V, E)$ over alphabet $[k]$ we reduce it to the following tester over $2^k \cdot |V|$ Boolean variables representing functions $f_v : \{0,1\}^k \to \{-1,1\}$ for all $v \in V$. The tester chooses an edge $(u,v) \in E$ uniformly at random and then applies the Håstad test with parameter δ to the average of f_u^{odd} and $f_v^{\text{odd}} \circ \sigma_{u\to v}$ where $\sigma_{u\to v} : [k] \to [k]$ is the permutation constraint on the edge (u,v) and for $x \in \{0,1\}^k$ we define $(\sigma_{u\to v}(x))_j = x_{\sigma_{u\to v}^{-1}(j)}$.

   (a) Show that completeness still works (and even better): if $\text{val}(G) \geq 1-\lambda$ then the resulting 3-linear CSP has value at least $1-\lambda-\delta$.

   (b) Show that soundness does not work: no matter what G is, the resulting 3-linear CSP has an assignment with value at least $5/8 - \delta/4$.

2. **No influential coordinates in the Håstad$_2$ test:** Give an example of a function $\pi : [k] \to [\ell]$ and balanced functions $f : \{0,1\}^k \to \{-1,1\}$, $g : \{0,1\}^\ell \to \{-1,1\}$ on which the Håstad$_2$ test passes with probability significantly greater than $1/2$ yet f has no influential coordinate.

3. **Close functions and concentration:** We say that f is ε-concentrated on a family $\mathcal{S}$ if $\sum_{S\notin\mathcal{S}} \hat{f}(S)^2 \leq \varepsilon$. Show that if $\|f-g\|_2^2 \leq \varepsilon$ and g is ε-concentrated on $\mathcal{S}$ then f is 4ε-concentrated on $\mathcal{S}$.