

# MATH-GA 2830 ADVANCED TOPICS IN APPLIED MATH: TURBULENT DYNAMICAL SYSTEMS

BY PROFESSOR ANDREW J MAJDA

*Time: Thursday, 1:15–3:15, Room 905; First Class: Thursday, September 8, 2016*

## COURSE DESCRIPTION:

Turbulent dynamical systems are ubiquitous complex systems in geoscience and engineering and are characterized by a large dimensional phase space and a large dimension of strong instabilities which transfer energy throughout the system. They also occur in neural and material sciences. Key mathematical issues are their basic mathematical structural properties and qualitative features, their statistical prediction and uncertainty quantification (UQ), their data assimilation, and coping with the inevitable model errors that arise in approximating such complex systems. These model errors arise through both the curse of small ensemble size for large systems and the lack of physical understanding. This is a research expository course on the applied mathematics of turbulent dynamical systems through the paradigm of modern applied mathematics involving the blending of rigorous mathematical theory, qualitative and quantitative modelling, and novel numerical procedures driven by the goal of understanding physical phenomena which are of central importance. The contents include the general mathematical framework and theory, instructive qualitative models, and concrete models from climate atmosphere ocean science. New statistical energy principles for general turbulent dynamical systems are discussed with applications, linear statistical response theory combined with information theory to cope with model errors, reduced low order models, and recent mathematical strategies for UQ in turbulent dynamical systems. Also recent mathematical strategies for online data assimilation of turbulent dynamical systems as well as rigorous results are briefly surveyed. Accessible open problems are often mentioned. This research expository book is the first of its kind to discuss these important issues from a modern applied mathematics perspective.

**Audience:** The course should be interesting for graduate students, and postdocs in pure and applied mathematics, physics, engineering, and climate, atmosphere, ocean science interested in turbulent dynamical systems as well as other complex systems.

## Course Organization:

- **Introductory Materials:**
  - Chapter 1, 2, 4, 6, 7, 8 from *Nonlinear Dynamics and Statistical Theories for Basic Geophysical Flow*, by A. J. Majda and X. Wang, Cambridge Press 2006;
  - *Introduction to Turbulent Dynamical Systems for Complex Systems*, *Frontiers in Applied Dynamical Systems*, by A. J. Majda, 85 pages, Springer 2016.
- **Style:** A lecture reading course with active participation of registered students and active discussion led by Di Qi, my fifth year PhD student.
- **Prerequisite:** Elementary background in ODEs and PDEs.
- **Grading:** All active registered participants will pass the course, no exams or problem set homework.