

HOMEWORK 9
MATH-UA 0248-001 THEORY OF NUMBERS

due on Nov, 30, 2020

1. Express the rational number $\frac{71}{55}$ as a finite simple continued fraction.
2. Compute the convergents of the simple continued fraction $[0; 2, 4, 1, 8, 2]$.
3. If $C_k = \frac{p_k}{q_k}$ is the k -th convergent of the simple continued fraction $[a_0; a_2, \dots, a_n]$, show that $q_k \geq 2^{(k-1)/2}$. (Hint: Observe that $q_k = a_k q_{k-1} + q_{k-2} \geq 2q_{k-2}$.)
4. For any positive integer n , show that $\sqrt{n^2 + 1} = [n; \overline{2n}]$. (Hint: notice that

$$n + \sqrt{n^2 + 1} = 2n + (\sqrt{n^2 + 1} - n) = 2n + \frac{1}{n + \sqrt{n^2 + 1}}.)$$