

HOMEWORK 6
MATH-UA 0248-001 THEORY OF NUMBERS

due on Nov, 2, 2020

1. Determine all the primitive roots of the prime 17.
2. If $ab \equiv r \pmod{p}$, where r is a quadratic residue of the odd prime p , prove that a and b are both quadratic residues of p or both nonresidues mod p .
3. If a and b are both quadratic residues of the odd prime p or both nonresidues, show that the congruence $ax^2 \equiv b \pmod{p}$ has a solution. (Hint: multiply by a' with $aa' \equiv 1 \pmod{p}$).
4. Evaluate the following Legendre symbols:
 - (a) $\left(\frac{71}{73}\right)$;
 - (b) $\left(\frac{3658}{12703}\right)$ (Hint: $3658 = 2 \cdot 31 \cdot 39$ and 12703 is prime).