HOMEWORK IV
THEORY OF NUMBERS
due on October, 12, 2020

1. Prove that there are infinitely many pairs of integers m and n with $\sigma(m^2) = \sigma(n^2)$. (Hint: look at $m = 5k, n = 4k$ with $(k, 10) = 1$).

2. Show that if $2^k - 1$ is a prime, then $n = 2^{k-1}(2^k - 1)$ satisfies the equation $\sigma(n) = 2n$.

3. Let a be a natural number whose last digit is an element in the set $\{1, 3, 7, 9\}$. Prove that the last two digits of a^{41} are the same as those of a. (hint: use Euler's theorem to establish that $a^{41} \equiv a$ $(mod\ 100)$.)

4. Consider the congruence $x^2 - 1 \equiv 0 \pmod 8$. How many solutions does it have with $0 \leq x < 8$?