

**HOMEWORK IV**  
**THEORY OF NUMBERS**

due on October, 5, 2020

1. Solve each of the following sets of simultaneous congruences:
  - (a)  $x \equiv 1 \pmod{3}$ ,  $x \equiv 2 \pmod{5}$ ,  $x \equiv 3 \pmod{7}$ ;
  - (b)  $2x \equiv 1 \pmod{5}$ ,  $3x \equiv 9 \pmod{6}$ ,  $4x \equiv 1 \pmod{7}$ ,  $5x \equiv 9 \pmod{11}$ .
2. Compute the following values of the Euler's Phi Function:
  - (a)  $\phi(120)$ ;
  - (b)  $\phi(10^k)$ , where  $k$  is a positive integer.
3. If  $p$  and  $q$  are distinct odd primes, prove  $2^{pq+1} \equiv 2^{p+q} \pmod{pq}$ .
4. Suppose that  $p_1, p_2, \dots, p_r$  are distinct primes that divide  $n$ . Show that the following formula for  $\phi(n)$  is correct.

$$\phi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right) \dots \left(1 - \frac{1}{p_r}\right).$$