**HOMEWORK I**
**THEORY OF NUMBERS**
due on September, 14, 2017, by 3.30pm
submit by email to pirutka@cims.nyu.edu

1. It is generally believed that infinitely many primes have the form $N^2 + 1$, although no one knows for sure. Do you think there are infinitely many primes of the form $N^2 - 1$?

2. Find a formula for all points on the hyperbola $x^2 - y^2 = 1$ whose coordinates are rational numbers. (Hint. Take the line through the point $(-1, 0)$ having rational slope $k$ and find a formula in terms of $k$ for the second point where the line intersects the hyperbola.)

3. Prove or disprove: if $a \mid (b + c)$, then either $a \mid b$ or $a \mid c$.

4. Determine the remainder of the division of

   (a) 2021 by 14;

   (b) $2021n + 1$ by 43, where $n$ is a positive integer.