MIDTERM: SOME PROBLEMS FROM PREVIOUS YEARS

1. Find all integers x, y satisfying the following equation:

$$98x + 68y = 2.$$

2. Solve the system of congruence equation:

$$\begin{cases} 3x \equiv 1 \pmod{23} \\ 2x \equiv 9 \pmod{17} \end{cases}$$

3. What is the remainder of 2^{124} when divided by 41?

4. What are all the primes $p < 14$ for which $x^{12} \equiv 1 \pmod{p}$ is valid for all x such that $(x, p) = 1$?

5. Which of the following functions is multiplicative? Why?

   (a) $\sigma(n)$ (the sum of divisors function);

   (b) $\phi(n)^2$;

   (c) $\phi(n) + 1$?

6. Let (x, y, z) be a primitive Pythagorean triple. Is it possible that

$$x \equiv 2 \pmod{8}? \text{ Why?}$$

7. Let p, q be distinct prime numbers, let α, β be positive integers and let $n = p^\alpha q^\beta$. Assume that the product $\alpha \cdot \beta$ is even. Is it possible that the number of divisors $\tau(n)$ of n is divisible by four: $4 \mid \tau(n)$? If your answer is 'yes', provide an example. If your answer is 'no', provide an explanation.