Homework 5 (circle method). Due by May, 2.

1. Assume every large integer is the sum of three primes. Prove every large even integer is the sum of two primes. Conversely, show if every large even integer is the sum of two primes, every large integer is the sum of three primes.

2. Show that
$$r_{1,s}(N) = \binom{N-1}{s-1} = \frac{N^{s-1}}{(s-1)!} + O(N^{s-2}).$$
Show that this bound is consistent with the Hardy-Littlewood asymptotic formula for $k = 1$.

3. Let $f(x)$ be a polynomial of degree $k \geq 2$ with integral coefficients and let
$$S_f(q, a) = \sum_{r=1}^{q} e(\frac{af(r)}{q}).$$
Frove that if $(q, r) = 1$, then
$$S_f(qr, ar + bq) = S_f(q, a)S_f(r, b).$$