

Homework 3. Due by March, 7.

1. Show that if $0 < a < 1$, then

$$\frac{1}{2\pi i} \int_{b-iT}^{b+iT} \frac{a^s ds}{s} = \mathcal{O}\left(\frac{a^b}{T|\log(a)|}\right).$$

2. Let $k > 0$ be an integer. Prove that

$$\int_2^x \frac{dt}{\log t} = \frac{x}{\log x} + \frac{1!x}{\log^2 x} + \dots + \frac{(k-1)!x}{\log^k x} + \mathcal{O}\left(\frac{x}{\log^{k+1} x}\right)$$

3. Let $T(x) = \sum_{n \geq 1} \Lambda(n)[x/n]$. Show that $T(x) = \sum_{n \leq x} \log n$.

4. Compute $\zeta(0)$.