

Homework 2. Due by February, 21.

1. Dirichlet theorem implies the following statement: if h and k are any two integers with $(h, k) = 1$, then there exists at least one prime number of the form $kn + h$. Prove that this statement also implies Dirichlet's theorem.
2. Prove that the function $f(t) = e^{-\pi t^2}$ is its own Fourier transform.
3. Let $\theta(t) = \sum_{n \in \mathbb{Z}} e^{-\pi n^2 t}$. For $t > 0$ prove that

$$\theta(t) = \frac{1}{\sqrt{t}} \theta\left(\frac{1}{t}\right).$$

4. Compute: a) $\Gamma\left(\frac{1}{2}\right)$; b) $\xi(1)$; c) $\xi(0)$.