Homework 1. Due by February, 7.

1. Let $S = \{1, 11, 21, 31, \ldots\}$ be the set of positive integers which have the last digit 1 when written in base 10. Prove that S has a Dirichlet density (in the set of all integers $\{1, 2, 3, \ldots\}$), and compute it.

2. Show that
$$\frac{\zeta'(s)}{\zeta(s)} = -\sum_p \sum_{m \geq 1} \frac{log\, p}{p^{ms}}$$
for $Re(s) > 1$, where $\zeta(s)$ is the Riemann zeta function.

3. Determine all Dirichlet characters modulo 8 and modulo 12.

4. Let χ be a Dirichlet character modulo m, $\chi(2) \neq 0$. Show that
$$L(s, \chi) = (1 - 2^{-s}\chi(2))^{-1} \sum_{n=0}^{\infty} \frac{\chi(2n+1)}{(2n+1)^s}.$$