HOMEWORK 7 MATH-GA 2350.001 DIFFERENTIAL GEOMETRY I

(part 1 is due by November, 21, 2016)

1. Let ω be a differential form on $\mathbb{R}^n$ given by:

$$\omega = \sum_{i=1}^{n} (-1)^{i-1} x_i dx_1 \wedge \ldots \widehat{dx_i} \ldots \wedge dx_n.$$

   (a) Compute $d\omega$.

   (b) Let $A : \mathbb{R}^n \to \mathbb{R}^n$ be a linear map. Compute $A^*\omega$.

2. Let U be an open in $E = \mathbb{R}^n$. Let $f : U \to \mathbb{R}$ be a smooth function, $X = \sum_{i=1}^n X_i \frac{\partial}{\partial x_i} \in \mathfrak{X}(U)$ be a vector field on U. Define: $grad\, f = \nabla f$ the vector field given by:

$$x \mapsto \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} \frac{\partial}{\partial x_i}$$

and $div\, X$ the function

$$x \mapsto \sum_{i=1}^{n} \frac{\partial X_i}{\partial x_i}.$$

Show that

   (a) $\mathcal{L}_X(dx_1 \wedge \ldots \wedge dx_n) = (div\, X) dx_1 \wedge \ldots \wedge dx_n$;

   (b) $div(fX) = f\, div X + \langle grad\, f, X\rangle$.