HOMEWORK 5 MATH-GA 2350.001 DIFFERENTIAL GEOMETRY I

(due by November, 7, 2016)

1. Show that a (connected) topological group is generated (as a group) by any neighborhood of its neutral element. Deduce that if G is a (connected) Lie group, then $exp(\mathfrak{g})$ generates G.

2. Let G be a smooth manifold and assume that G has a group structure such that the map $(x, y) \mapsto xy$ is smooth. Show that G is a Lie group. (Hint: consider the map $(x, y) \mapsto (x, xy)$ in a neighborhood of (e, e).)

3. Show that the vector space $\mathbb{R}^3$ with the vector product $\wedge$ is a Lie algebra. Consider the vector fields X, Y, Z on $\mathbb{R}^3$:

$$X = z\frac{\partial}{\partial y} - y\frac{\partial}{\partial z},\ Y = x\frac{\partial}{\partial z} - z\frac{\partial}{\partial x},\ Z = y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y}.$$

   Show that they generate a Lie subalgebra, in the Lie algebra $\mathfrak{X}(\mathbb{R}^3)$, isomorphic to $\mathbb{R}^3$ with the product structure $\wedge$.