## HOMEWORK 4 MATH-GA 2350.001 DIFFERENTIAL GEOMETRY I (due by October, 24, 2016)

- 1. Show that on  $\mathbb{R}^2$  the vector fields  $y^2 \frac{\partial}{\partial x}$  and  $x^2 \frac{\partial}{\partial y}$  are complete, but the vector field  $y^2 \frac{\partial}{\partial x} + x^2 \frac{\partial}{\partial y}$  is not complete.
- 2. Let  $a \in \mathbb{R}^n$ . Let  $X_a$  be the vector field defined by  $X_a(y) = a y$ .
  - (a) Compute the flow of  $X_a$ ;
  - (b) If  $b \in X_b$ , compute  $[X_a, X_b]$  using the flows of  $X_a$  and  $X_b$ .
- 3. Find global noncommuting flows on  $S^2$ .