HOMEWORK 3 MATH-GA 2350.001 DIFFERENTIAL GEOMETRY I

(due by October, 17, 2016)

1. Consider the usual charts on $\mathbb{R}P^2$:

$$[x, y, z] \mapsto (u_1, u_2) = (x/z, y/z) \text{ on } U_z = \{z \neq 0\};$$

$$[x, y, z] \mapsto (v_1, v_2) = (x/y, z/y) \text{ on } U_y = \{y \neq 0\};$$

$$[x, y, z] \mapsto (w_1, w_2) = (y/x, z/x) \text{ on } U_x = \{x \neq 0\}.$$

   Show that there is a vector field on $\mathbb{R}P^2$ which in the last coordinate chart has the following expression:

$$w_1 \frac{\partial}{\partial w_1} - w_2 \frac{\partial}{\partial w_2}.$$

   What are the expressions of this vector field in the other two charts?

2. Construct a smooth vector field on the sphere S^2 that vanishes only at one point.

3. Let M, N be smooth manifolds and let $f : M \to N$ be a smooth submersion. Show that for any smooth vector field Y on N there exists a smooth vector field X on M such that

$$\forall x \in M \quad T_x f(X(x)) = Y(f(x)).$$