

HOMEWORK VIII
MATH-UA 0248-001 THEORY OF NUMBERS

due on Nov, 10, 2017

1. Solve the following quadratic congruence equation:

$$3x^2 + 9x + 17 \equiv 0 \pmod{13}.$$

2. If $ab \equiv r \pmod{p}$, where r is a quadratic residue of the odd prime p , prove that a and b are both quadratic residues of p or both nonresidues mod p .
3. If a and b are both quadratic residues of the odd prime p or both nonresidues, show that the congruence $ax^2 \equiv b \pmod{p}$ has a solution. (Hint: multiply by a' with $aa' \equiv 1 \pmod{p}$).
4. Use Gauss' Lemma to compute
- (a) $\left(\frac{7}{13}\right)$;
 - (b) $\left(\frac{6}{31}\right)$.
5. If $p \equiv 1 \pmod{8}$, show that $p \mid 2^{\frac{p-1}{2}} - 1$ (hint: use the expression of $\left(\frac{2}{p}\right)$).