**HOMEWORK VI**
**MATH-UA 0248-001 THEORY OF NUMBERS**
due on October, 27, 2017

1. Prove that there are infinitely many pairs of integers $m$ and $n$ with $\sigma(m^2) = \sigma(n^2)$. (Hint: look at $m = 5k, n = 4k$ with $(k, 10) = 1$).

2. Show that if $2^k - 1$ is a prime, then $n = 2^{k-1}(2^k - 1)$ satisfies the equation $\sigma(n) = 2n$.

3. For each positive integer $n$ show that
$$\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0.$$

4. Let $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_r^{\alpha_r}$ be the prime factorization of the integer $n > 1$. If $f$ is a multiplicative function, prove that
$$\sum_{d|n} \mu(d) f(d) = (1 - f(p_1))(1 - f(p_2)) \dots (1 - f(p_r)).$$
(Hint: is the function $F(n) = \sum_{d|n} \mu(d) f(d)$ multiplicative?)

5. Let $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_r^{\alpha_r}$ be the prime factorization of the integer $n > 1$. Prove that
   (a) $\sum_{d|n} \mu(d)\sigma(d) = (-1)^r p_1 p_2 \dots p_r$;
   (b) $\sum_{d|n} \mu(d)/d = (1 - \frac{1}{p_1})(1 - \frac{1}{p_2}) \dots (1 - \frac{1}{p_r})$.