HOMEWORK III
MATH-UA 0248-001 THEORY OF NUMBERS
due on September, 29, 2017

1. Prove each of the following assertions:
   (a) If $a \equiv b \pmod{n}$ and $m \mid n$ then $a \equiv b \pmod{m}$.
   (b) If If $a \equiv b \pmod{n}$ and $c > 0$, then If $ca \equiv cb \pmod{cn}$.
2. Find the remainders when 2^{50} and 41^{65} are divided by 7.
3. Prove that for any positive integer n, the following congruences hold:
   (a) $2^{2n} \equiv 1 \pmod{3}$.
   (b) $2^{3n} \equiv 1 \pmod{7}$.
   (c) $2^{4n} \equiv 1 \pmod{15}$.
4. Prove that, if p is an odd prime, then
   (a) $1^{p-1} + 2^{p-1} + 3^{p-1} + \ldots + (p-1)^{p-1} \equiv -1 \pmod{p}$;
   (b) $1^p + 2^p + 3^p + \ldots + (p-1)^p \equiv 0 \pmod{p}$.
5. Establish that if a is an odd integer, then
$$a^{2^n} \equiv 1 \pmod{2^{n+2}}.$$
   (Hint: proceed by induction on n)