HOMEWORK X
MATH-UA 0248-001 THEORY OF NUMBERS

due on Dec, 1, 2017

1. Show that if $n \equiv 3$ or $6 \pmod 9$, then n cannot be represented as a sum of two squares.

2. Write the integer $39690 = 2 \cdot 3^4 \cdot 5 \cdot 7^2$ as a sum of two squares.

3. Express the rational number $\frac{71}{55}$ as a finite simple continued fraction.

4. Compute the convergents of the simple continued fraction $[0; 2, 4, 1, 8, 2]$.

5. If $C_k = \frac{p_k}{q_k}$ is the k-th convergent of the simple continued fraction $[a_0; a_2, \ldots a_n]$, show that $q_k \geq 2^{(k-1)/2}$. (Hint: Observe that $q_k = a_k q_{k-1} + q_{k-2} \geq 2q_{k-2}$.)

6. For any positive integer n, show that $\sqrt{n^2+1} = [n; \overline{2n}]$. (Hint: notice that

$$n + \sqrt{n^2+1} = 2n + (\sqrt{n^2+1} - n) = 2n + \frac{1}{n + \sqrt{n^2+1}}.$$

7. Find the fundamental solution of the equation $x^2 - 41y^2 = 1$. (Hint: $\sqrt{41} = [6, \overline{2, 2, 12}]$.)