HOMEWORK I
MATH-UA 0248-001 THEORY OF NUMBERS
due on September, 15, 2017

1. It is generally believed that infinitely many primes have the form $N^2 + 1$, although no one knows for sure. Do you think there are infinitely many primes of the form $N^2 - 1$?

2. Show that for a Pythagorean triple (X, Y, Z) exactly one of the following holds:

   (a) all X, Y, Z are multiples of 3;

   (b) X is a multiple of 3 and Y is not;

   (c) Y is a multiple of 3 and X is not.

3. Find a formula for all points on the hyperbola $x^2 - y^2 = 1$ whose coordinates are rational numbers. (Hint. Take the line through the point $(-1, 0)$ having rational slope k and find a formula in terms of k for the second point where the line intersects the hyperbola.)

4. Fermat's Last Theorem states that if $n \in \mathbb{N}$ and $n \geq 3$, the equation

$$X^n + Y^n = Z^n$$

   has no solutions in nonzero integers X, Y, Z. Make use of this theorem to prove that $\sqrt[n]{2} \notin \mathbb{Q}$ where $n \in \mathbb{N}$ and $n \geq 3$.