

MATH-GA 2210.001: Homework Number theory 5

1. If $K = \mathbb{Q}(\sqrt{d})$, with $d \in \mathbb{Z} \setminus \{0, 1\}$ without square factors, then $\text{disc}(\mathcal{O}_K) = d$ or $4d$, corresponding to $d \equiv 1 \pmod{4}$ or not.
2. Determine $Cl(\mathbb{Z}[\sqrt{-14}])$:
 - (a) Show that any nonzero ideal of $A = \mathbb{Z}[\sqrt{-14}]$ is equivalent to an ideal containing an integer $1 \leq N \leq 4$.
 - (b) Determine all the ideals of A containing 1, 2, 3 and 4.
 - (c) Considering small numbers represented by a form $x^2 + 14y^2$, show that $Cl(A)$ is cyclic of order 4.