

MATH-GA 2210.001: Homework Number theory 4

1. Let $\alpha = \frac{1+\sqrt{-19}}{2}$ and $A = \mathbb{Z}[\alpha]$.
 - (a) Show that any ideal of A is equivalent to an ideal containing 1 or 2.
 - (b) Deduce that A is principal, but not euclidean.

2. Consider $\alpha = \sqrt{-5}$ and $A = \mathbb{Z}[\alpha]$.
 - (a) Show that any ideal of A is principal or equivalent to an ideal $I = 2A + (\alpha - 1)A$.
 - (b) Show that I is not principal, deduce that $|Cl(A)| = 2$.
 - (c) Show that $I^2 = (2)$ and deduce that $Cl(A) = \mathbb{Z}/2\mathbb{Z}$.
 - (d) Deduce that if J is an ideal of A , then either J or IJ is principal.

3. Let $\alpha = \sqrt{-3}$ and $A = \mathbb{Z}[\alpha]$.
 - (a) Show that any ideal of A is principal or equivalent to an ideal $I = 2A + (\alpha - 1)A$.
 - (b) Show that I is not principal, deduce that $|Cl(A)| = 2$.
 - (c) Show that $I^2 = 2I$.
 - (d) Deduce that $Cl(A)$ is not a group and describe its composition law.
 - (e) Deduce that if J is a nonzero ideal of A , then IJ is never principal.