MATH-GA 2210.001: Homework Algebraic Number Theory 2

1. Which of the following rings are Dedekind:

$$\mathbb{Z}[\sqrt{5}],\ \mathbb{Z}[\frac{1+\sqrt{5}}{2}],\ \mathbb{R}[x,y,z]/(x^2+y^2+z)?$$

2. ("moving lemma") Let I be a nonzero ideal of the Dedekind ring A with field of fractions K. Show that there exists $a \in K^*$ such that aI is an ideal of A, prime to I (i.e. aI and I have no common prime factors).

3. Let A be an integral ring, such that any nonzero ideal has a unique decomposition (up to a permutation) as product of prime ideals. Show that A is a Dedekind ring.

4. Let $\alpha^3 = 2$ and $L = \mathbb{Q}(\alpha)$. Find the decomposition of the ideal (5) in the ring of integers $\mathcal{O}_L$ of L (you may use that $\mathcal{O}_L = \mathbb{Z}[\alpha]$).