MATH-GA 2210.001: Homework Algebraic Number Theory 1

1. Let $A \subset B$ be an inclusion of rings. Assume that B is noetherian. Is it true that A is noetherian?

2. Let A be a Dedekind ring and let I, J be nonzero ideals of A. Express, in terms of decomposition of I and J as a product of powers of prime ideals, the decomposition of
$$I \cap J, IJ, I + J.$$
Deduce that if $\mathfrak{p}_1, \ldots, \mathfrak{p}_r$ are distinct prime ideals of A, then $\prod_{i=1}^r \mathfrak{p}_i = \cap_{i=1}^r \mathfrak{p}_i$.

3. Let I be a nonzero ideal of a Dedekind ring A and assume $I \neq A$. Show that I is generated by two elements.

4. Show that a Dedekind ring with finitely many prime ideals is principal.