MATH-GA 2210.001: Homework Local Fields 2

1. Show that in $\mathbb{Q}_2$
$$1 + 2 + \ldots + 2^n + \ldots$$
converges to -1.

2. Let K be a filed and $|\ |$ a nonarchimedean absolute value on K.
   (a) Recall that an open disk centred at a of radius r is defined as $B(a, r) = \{x \in K, |x - a| < r\}$. Show that $B(a, r) = B(b, r)$ for any $b \in D(a, r)$. Deduce that if two disks meet, then the large disk contains the smaller.
   (b) Assume K is complete. Show that the series $\sum a_n$ converges iff $a_n \to 0$.

3. For which $a \in \mathbb{Z}$ the equation $7x^2 = a$ is solvable in $\mathbb{Z}_7$? For which $a \in \mathbb{Q}$ is it solvable in $\mathbb{Q}_7$?

4. (a) Show that $(x^2 - 2)(x^2 - 17)(x^2 - 34)$ has a root in $\mathbb{Z}_p$ for every p.
   (b) Show that $5x^3 - 7x^2 + 3x + 6$ has a root α in $\mathbb{Z}_7$ with $|\alpha - 1|_7 < 1$. Find $a \in \mathbb{Z}$ such that $|\alpha - a|_7 \leq 7^{-4}$.