1. Determine a) $H\left(\frac{2}{3}\right)$; b) $H\left(\frac{1+\sqrt{-5}}{2}\right)$.

2. Let $K$ be a number field and let $L/K$ be a finite extension. Let $p \subset \mathcal{O}_K$ be a prime corresponding to an absolute value $v$ on $K$. Write $p\mathcal{O}_L = \prod q^e_w$ decomposition into prime ideals and set $f_w$ the degree $[\mathcal{O}_L/q_w : \mathcal{O}_K/p]$.

(a) Show that $\# \mathcal{O}_L/p = (\# \mathcal{O}_K/p)^{[L:K]}$ (one can use that $\mathcal{O}_K = \mathbb{Z}e_1 \oplus \ldots \oplus \mathbb{Z}e_n$ for $n = [K : \mathbb{Q}]$ and $e_i \in \mathcal{O}_K$, $i = 1 \ldots n$, similairly for $L$.)

(b) Deduce that $[L : K] = \sum e_wf_w$.

(c) If $x \in \mathcal{O}_K$ show that $ord_{q_w}x = e_word_p(x)$. Deduce that $|x|_w = |x|^{ewf_w}_v$.

(d) Deduce that for $x_0, \ldots x_n \in K$ one has $\prod_{w|v} max_i |x_i|_w = max_i |x_i|^{[L:K]}_v$.

(e) Deduce that for $P \in \mathbb{P}^n(K)$ one has $H_L(P) = H_K(P)^{[L:K]}$.