MATH-GA 2150.001: Homework 1 : some answers

1 $J = \langle x^2 + y^2 - 1, y - 1\rangle \subset k[x, y]$.

a $V(J) = \{(0, 1)\}$.

b $f = x$ works

c $I(V(J)) = (x, y - 1)$.

3f i. $X = V(y, y^2 - xz) \subset \mathbb{A}^3_k$; the irreducible components are $\{y = 0, x = 0\}$ and $\{y = 0, x = 0\}$.

ii. $X = V(x(y - x^2 + 1), y(y - x^2 + 1)) \subset \mathbb{A}^2_k$; the irreducible components are $\{x = 0, y = 0\}$ and $\{y - x^2 + 1 = 0.\}$

iii. $X = V(x^2) \subset \mathbb{A}^2_k$, $X = \{x = 0\}$ is irreducible.

4 a $X = V(x^2y, (x-1)(y+1)^2), I(X) = (x(x-1), xy, (x-1)(y+1), y(y+1))$,

b $X = V(y^2 + x^2y - x^2), I(X) = (y^2 + x^2y - x^2)$,

c $X = V(z - xy, y^2 + xz - x^2), I(X =)(z - xy, y^2 + xz - x^2)$.