Algebra I. Homework 8. Due on November 19, 2020.

1. Let $\mathbb{F}_p$ be a finite field with p elements and let $K = \mathbb{F}_p(x, y)$. Show that $K(x^{1/p}, y^{1/p})$ is a finite extension of K which is not simple.

2. Let $\zeta_n = exp(2\pi i/n)$. Show that $\mathbb{Q}(\zeta_n, \zeta_m) = \mathbb{Q}(\zeta_{lcm(n,m)})$, where $lcm(n, m)$ denotes the least common multiple of m and n.

3. Compute the number of field morphisms $\mathbb{Q}(\sqrt[3]{2}, j) \to \mathbb{C}$ where $j^3 = 1$ and $j \neq 1$ is a cubic root of unity.

4. Let K be a field of characteristic $p > 0$ and let $K^p \subset K$ be the subfield of p-powers in K. Could K be a finite extension of K^p? Could it be infinite?