Algebra I. Homework 7. Due on November 12, 2020.

1. Find irreducible components of the following varieties :

   (a) $V(y, y^2 - xz) \subset \mathbb{A}^3_k$;

   (b) $V(x(y - x^2 + 1), y(y - x^2 + 1)) \subset \mathbb{A}^2_k$.

   (c) $V(x^2) \subset \mathbb{A}^2_k$

2. Show that the set of lines in $\mathbb{P}^2_k$ form a projective space.

3. Show that the map $\mathbb{P}^n_k \times \mathbb{P}^m_k \to \mathbb{P}^{mn+m+n}_k$ given by

$$((x_0 : x_1 \ldots : x_n), (y_0 : y_1 \ldots : y_m)) \mapsto (x_i y_j)_{0 \le i \le n, 0 \le j \le m}$$

   is a bijections between $\mathbb{P}^n_k \times \mathbb{P}^m_k$ and a closed set (in Zariski topology) of $\mathbb{P}^{mn+m+n}_k$.