Algebra I. Homework 4. Due on October 8, 2020.

1. Prove or disprove: a subring of a Noetherian ring is Noetherian.
2. Let p be a prime number. Show that the polynomial $1+x+\ldots+x^{p-1}$ in $\mathbb{Q}[x]$ is irreducible.
3. Prove that $f(x) = x^4 + 4x + 1 \in \mathbb{Q}[x]$ is irreducible.
4. Let $R = \mathbb{C}[x, y]$ and let I be the ideal of R generated by two elements (x, y). Prove or disprove: I is a free R-module.