Algebra I. Homework 3. Due on October 1, 2020.

1. Let I, J be ideals in a ring R.
   (a) Show that $I \cup J$ is not necessarily an ideal in R;
   (b) Show that $I + J = \{x \in R | x = i + j, i \in I, j \in J\}$ is an ideal in R.

2. Let R be a ring. Let $I \subset R$ be an ideal such that any element of $R \setminus I$ (i.e. an element of R which is not in I) is invertible. Prove that I is a maximal ideal.

3. Prove or disprove: if R is a principal ring, then $R[x]$ is principal.

4. (a) Give an example of a noetherian ring which is not factorial (hint: look at $\mathbb{Z}[i\sqrt{5}]$);
   (b) Give an example of a factorial ring which is not noetherian (hint: look at $\mathbb{R}[x_n]_{n\in\mathbb{N}}$, you can use the theorem announced in Lecture 4 that if R is factorial, then $R[x]$ is factorial)